A MODEL OF CONFLICT, APPROPRIATION AND PRODUCTION IN A TWO-SECTOR ECONOMY

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by

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ABSTRACT

This paper presents a model of conflict in an economy characterized by two sectors. In a first sector labelled as contested sector two agents struggle in order to appropriate the maximum possible fraction of a contestable output. In a second sector, the uncontested sector, each agent holds secure property rights over the production of some goods. Both agents maximize an income function which can be defined as a function of contributions of both sectors. Results show that the degree of returns in the uncontested sector is a powerful force which countervails the impact of destructive and unproductive interaction in the contested sector. Eventually, in the presence of a redistributive government both total production and total welfare can be raised thanks to a superior productivity. However, a scenario characterized by the existence of a redistributive government appears to be less peaceful than a 'minimum government' scenario.

KEYWORDS: Conflict, Productive and Unproductive Activities, Butter and Guns, Appropriation, entrepreneurship, minimum government, redistributive government.

JEL CLASSIFICATION: D74, D20, F51, H56, O17.

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Introduction

This paper is intended to be a contribution to the theoretical economic analysis of conflict. A conflict can be described as «a destructive interaction which involves strategic interdependent decisions in the presence of coercion and anarchy». In many general equilibrium models following Hirshleifer (1988), 1 a contestable output falls into a common pool available for seizure and appropriation. The chosen levels of resources invested exclusively in productive or unproductive activities determine the social outcome of a conflict. Hirshleifer’s seminal work and following contributions analyse a simplified economy where all productive activities are under the threat of violent appropriation. However, in reality, agents involved in a conflict have some income and wealth secure from appropriation. Hence, there must be a relationship between the choice of resources to be allocated to conflict and the choice of resources to be allocated in a secure production. Then, broadly speaking in an extremely simplified economy, it would be possible to consider two sectors. In a first sector, each agent holds secure property rights over the production of some goods. Such secure property rights assure the holder of a secure level of production and then of income stream. In a second sector, agents struggle in order to appropriate the maximum possible fraction of a contestable output. In the continuation of this work, I shall label the first sector as uncontested sector and the latter as contested sector.

Several reasons can be advanced to distinguish between uncontested and contested sectors. First, there could be institutional factors protecting contract and property rights. In fact, there could be sectors where enforcement of property rights can be more effective than others. In some cases, property rights can be effectively enforced thanks to government’s commitment. In some other cases, this needs not be related with the existence of a government capable to behave as monopolist of violence. Even stateless societies have developed informal institutions able to enforce property rights assignments. These informal structures were related to some specific factors as kin-ties, cheap and available information, reputation and social capital. These institutions in many cases are able to cope with the problem of management of common resources avoiding the “tragedy of commons”, (Collier and Gunning, 1999). Secondly, there could be geographical factors shielding some sectors from destructive conflicts and violent appropriation. On one hand, there could geographical obstacles making the struggle for appropriation less feasible. Instead, there are some fractions of territory more attractive than others because of their resources endowments and productive structures. This is verifiable when different warlords (or states and rebel groups) fight over the appropriation and the control of a territory. On one hand they fight and expend resources in an identified fraction of territory to appropriate a contested resource. On the other hand, they can be involved in productive activities on the fraction of territory whose govern is

completely secure. Finally, there are economic reasons shaping the preferences set of actors involved and spontaneously identifying a boundary between contested and uncontested sectors. Within a set of possible choices rational agents can be predicted to prefer activities which are supposed to guarantee higher monetary returns. Whenever the expected returns from appropriation and bloody rent-seeking are assumed to be greater than those attainable through investments in ordinary entrepreneurial businesses, a rational agent may divert its efforts and resources to them. Therefore, in such a scenario, some entrepreneurial activities can be interpreted as non-attractive from rational agents. Paradoxically, being less profitable and non-attractive some activities are more secure from appropriation. Of course, these three sources of distinction between contested and uncontested sector are strictly correlated and can often overlap.

A fitting example could be drawn from reality of many African developing countries which experience the sadly famous ‘resource curse’. In particular, according a paradigmatic sketch, the government and different warlords compete over the appropriation of rents flourishing from exports of natural resources. This leads to social unrest and violent competition. In fact, it is now fully acknowledged that emergence of civil wars is positively related with the exploitation of rents flourishing in some sectors (see among others Collier and Hoeffler 1998, Le Billon 2001a, De Soysa 2002, Fearon and Laitin 2003). Hence, in some cases the distinction between contested and uncontested is relatively simple. Studying the Angolan civil war Guidolin and La Ferrara (2007) explain their focus on the diamond sector, because “unlike oil production sites, which are located offshore and were removed from the fighting in the mainland, the activities of diamond extracting firms were located in areas very much at the heart of the conflict.” The latter statement is clear in this respect even if it should be reversed. It is not the diamond sector which is located at the heart of conflict but it is the conflict which is located at the heart of diamond sector. General empirical evidence is provided in Buhaug and Gates (2002) that show how localization of a civil war is positively related with the presence of natural resources. In particular, the authors studied the location of all battles thereby identifying the geographic extent of 265 civil conflicts over the period 1946-2000 finding a robust positive association between the occurrence of violent conflicts and natural resources location.

The distinction between contested and uncontested sectors opens questions about the design of economic policies able to cope with both the persistence of bloody conflicts and the emergence of welfare-enhancing institutions. In this respect, - albeit lacking strong theoretical underpinnings - Ross (2003) compares the cases of Nigeria and Indonesia. Among other factors, the author maintains that in Indonesia the governments have been committed to support agricultural and manufacturing sectors. Instead Nigerian governments focused upon exploitation of Oil sector thus undermining entrepreneurial activities in small manufacturing sector and agriculture. Instead, Indonesia avoided the crowding-out of productive sectors as manufacturing and agriculture. The reliance upon some contested sectors is also the case of other African developing countries descended to civil wars as – among

3 See also the account given by Omeje (2004).
others - Liberia, Uganda and Angola. (see respectively Johnston 2004, Deininger 2003, Le Billon 2001b, Malaquias 2001)

Hence, in the continuation of this work, I shall present a simplified economy characterized by two sectors labelled respectively as *contested* and *uncontested*. Two rational agents split their own positive resource endowment between two kinds of productive activities and unproductive activities. Beyond the classical ‘butter’ and ‘guns’ I shall label the productive investments in the uncontested sector ‘ice creams’. There is a productive asymmetry between the two sectors. That is, there is an uncontested sector characterized by decreasing returns to scale (DRS) and a contested sector characterized by constant returns to scale (CRS). The final allocation of resources between ‘butter’, ‘guns’ and ‘ice creams’ will depend upon exploitation of force.

To the best of my knowledge, within a growing literature on conflict theory there are very few papers analysing two sectors with three activities as two kinds of productive activities (secure production, contested production) and unproductive activities. Garfinkel and Skaperdas (2007) introduced the argument in a section of their survey on economics of conflict. In a two-agent world, the authors assumed that agents can produce butter, guns and an inferior substitute for butter, called ‘margarine’. The latter is assumed to be secure from appropriation by the rival. In the presence of perfectly enforced property rights over the production of butter, both agents would not have any incentive to produce margarine. Then, their model allows for two types of equilibria. In the first equilibrium agents only produce ‘margarine’ thus implying no allocation of resources to both ‘butter’ and ‘guns’. In a second kind of equilibrium, both parties produce positive quantities of guns and butter but no margarine. Different equilibria emerge in the presence of particular combination of a degree of decisiveness of the conflict and a productivity parameter. Whenever the degree of productivity for margarine is relatively high with respect to the decisiveness of violent conflict, agents are likely to invest only in the secure production of margarine.

More attention has been paid to economies characterized by two kinds of unproductive activities (defence and offence) and productive activities. This is the case of Grossman and Kim (1995), Rider (1999) and Panagariya and Shibata (2000) among others. The latter, models an arms rivalry between two small countries facing a constant probability of war. Countries produce arms and a consumption good that can be traded internationally whilst a defence good interpreted as a public good is non-traded. The main result of the article is that a subsidy flowing from one country to another can boost consumption and then increase total welfare. Rider (1999) develops a model with two goods and three activities (production, predation and defence) to show the impossibility of pure and uncontested exchange. In such a framework each agent is assumed to produce only one good.

This brief paper is simply designed. In a first section, a basic model is presented. In a second section, the impact of different variables and parameters upon total production and total welfare are studied. In a third section, under some simplifying assumptions, the model is enriched in order to analyse the interaction between a government and a rival group. Eventually, a brief comparison between
the two scenarios is presented. In the last section, results are summarized and some conclusions are presented.

A Basic Model

The world is made of two risk-neutral agents indexed by $i = 1, 2$. They interact simultaneously. Both agents have a positive resources endowment denoted by $R_i \in (0, \infty), i = 1, 2$. It can be divided into ‘guns’, ‘butter’ and ‘ice-creams’. By ‘guns’ I indicate any positive investments in unproductive activities of fighting. By ‘butter’ I indicate any positive investment in productive activities in the contested sector, whilst by ‘ice-creams’ I indicate any positive investments in productive activities in the unchallenged sector. The interaction between the two agents generates an equilibrium allocation of resources endowment among ‘guns’, ‘butter’ and ‘ice-creams’. To summarise formally it is possible to write the resources constraint as:

$$R_i = y_i + x_i + G_i, i = 1, 2$$

where $G_i$ denotes the level of ‘guns’, and $y$ and $x$ denote ‘ice-creams’ and ‘butter’ respectively. They are all assumed to be positive: $y_i \in (0, \infty), x_i \in (0, \infty), i = 1, 2$. In the contested sector, the contested joint product – indicated by $CY$ - can be described as a simple linear additive function:

$$CY = x_i + x_i = TR - G_i - y_i - G_i - y_i$$

where $TR = R_i + R_j$. For convenience assume $TR \in [1, \infty)$. This aggregate production function is characterized by constant returns to scale and constant elasticity of substitution. The outcome of the struggle is determined by means of an ordinary Contest Success Function\(^4\) (henceforth CSF for brevity) in its ratio form:

$$p_i(G_i, G_j) = \frac{G_i}{G_i + G_j}, i = 1, 2$$

The functional form adopted for CSF implies that the conflict is not decisive, namely it could be said that it exhibits constant returns to fighting. Equation (3) is differentiable and follows the conditions below:

\[
\begin{align*}
p_i + p_j &= 1 \\
p_i &= 0.5 \text{ at } G_i = G_j \quad \quad (3.1) \\
\partial p_i / \partial G_i > 0 &\quad \partial p_i / \partial G_j < 0 \\
\partial^2 p_i / \partial G_i < 0 &\quad \partial^2 p_i / \partial G_j > 0 
\end{align*}
\]

and then the outcome in the contested sector is given by:

\[S_i = p \left( G_i, G_j \right) \theta \]

Where \( \theta \in (0, 1) \) denotes a physical destruction parameter. It can be interpreted as an ex-ante perception of destructiveness of conflict. That is, a conflict is twice costly. On one hand the amount resources allocated to ‘guns’ do constitute a deadweight loss for society because the same amount of resources could be allocated to more productive activities. On the other hand, in the case of actual violent conflicts there is a fraction of resources physically destroyed. Given the analytical complexity, I shall assume for sake of simplicity that it is equal for both agents. As \( \theta \) increases, the conflict is perceived less and less destructive. Given conditions (3.1) the fraction of contestable output accruing to agent \( i \) is increasing in its own level of guns whereas it is decreasing in the opponent’s level of guns.

The uncontested sector is modelled as a traditional sector exhibiting decreasing returns to scale. Therefore, the production function is a standard intensive production function which exhibits decreasing returns to scale:

\[Y_i(y_i) = y_i^a; Y_j(y_j) = y_j^b\]

where \( y_i \) denotes the level of resources devoted to the uncontested production by agent \( i \) and \( a \in (0, 1) \) and \( b \in (0, 1) \) are the parameters capturing the degree of returns of scale for agent 1 and agent 2 respectively. Trivial to say that \( Y(0) = 0, Y(\infty) = \infty \), \( \partial Y / \partial y > 0, \partial^2 Y / \partial y^2 < 0 \), \( \partial Y_i / \partial a > 0 \Leftrightarrow y_i > 1, \partial Y_i / \partial b > 0 \Leftrightarrow y_i > 1 \). The level of production in the uncontested sector can be simply denoted through \( UY = Y_i + Y_j \).

Therefore, the final income of each agent can be described as a function of contributions of both sectors as \( W = f(Y, S) \). Eventually, each agent maximizes an objective function as:

\[W(Y_i, S_i) = Y_i + S_i, i = 1, 2\]

This kind of function can lead to ambiguous results. On one hand, an increase in the amount of ‘guns’ lowers the level of production. On the other hand, final wealth of each agent could be raised through positive investments in appropriative activities. Agents are assumed to be rational and to interact simultaneously à la Nash-Cournot. Therefore, treating the opponent’s choice as given each agent
maximizes (6). Under an ordinary process of maximization the equilibrium choices of ‘ice-creams’ are:

\[ y_i^* = \left( \frac{2a}{\theta} \right)^{\frac{1}{(1-a)}} \]  
(7.1)

\[ y_2^* = \left( \frac{2b}{\theta} \right)^{\frac{1}{(1-b)}} \]  
(7.2)

The equilibrium level of ‘ice-creams’ is increasing in the degree of returns to scale, \( \partial y_i^*/\partial a > 0, \partial y_2^*/\partial b > 0 \). Trivial to say that \( y_i^* = y_2^* \) for \( a = b \). Note also that the level of ‘ice-creams’ is decreasing in the destruction parameter \( \partial y_i^*/\partial \theta < 0 \). A smaller degree of destruction implies fewer resources are allocated to production in the uncontested sector. The equilibrium level of ‘guns’ is given by:

\[ G_i^* = G_2^* = G^* = \frac{TR}{4} - 2 \left( 2^{(2a-1)(1-a)} \left( \frac{a}{\theta} \right)^{\frac{1}{(1-a)}} \right) - 2 \left( 2^{(2b-1)(1-b)} \left( \frac{b}{\theta} \right)^{\frac{1}{(1-b)}} \right) \]  
(8)

A necessary and sufficient condition to have an equilibrium for the solutions shown in (7.1), (7.2) and (8) is \( TR > \left( 2a/\theta \right)^{\frac{1}{(1-a)}} + \left( 2b/\theta \right)^{\frac{1}{(1-b)}} \), namely \( TR > y_i^* + y_2^* \). Clearly, the latter condition always holds. Note that the level of guns is increasing in the destruction parameter, \( \partial G^*/\partial \theta > 0 \). Namely, the lower is the perceived potential destruction the higher is the investment in arms. Moreover it is clear that \( \partial G^*/\partial a < 0, \partial G^*/\partial b < 0 \). It is possible to compute the equilibrium level of ‘butter’ simply as:

\[ x_i^* = R_i - y_i^* - G_i^* = \]
\[ = \frac{3R_i - R_2}{4} - 3 \times 2 \left( 2^{(2a-1)(1-a)} \left( \frac{a}{\theta} \right)^{\frac{1}{(1-a)}} \right) + 2 \left( 2^{(2b-1)(1-b)} \left( \frac{b}{\theta} \right)^{\frac{1}{(1-b)}} \right) \]  
(9.1)

\[ x_2^* = R_2 - y_2^* - G_2^* = \]
\[ = \frac{3R_2 - R_i}{4} - 3 \times 2 \left( 2^{(2b-1)(1-b)} \left( \frac{b}{\theta} \right)^{\frac{1}{(1-b)}} \right) + 2 \left( 2^{(2a-1)(1-a)} \left( \frac{a}{\theta} \right)^{\frac{1}{(1-a)}} \right) \]  
(9.2)

And it is possible to show that the level of butter of each agent is decreasing in its degree of returns to scale and increasing in rival’s degree of return to scale; namely \( \partial x_i/\partial a < 0, \partial x_i/\partial b > 0, \partial x_2/\partial b < 0, \partial x_2/\partial a > 0 \). This means that as the degree of returns to scale increases each agent will prefer to allocate resources to the uncontested sector. That is, as the secure and uncontested sector becomes more productive (albeit still in the range of the DRS) the level of contested ‘butter’ decreases.
Of course, the level of butter of agent $i$ is increasing in its own initial endowment and decreasing in the endowment of the opponent, namely $\partial x^i / \partial R^j > 0, \partial x^i / \partial R^j < 0, i = 1,2, i \neq j$. Final incomes of both agents are then given by:

$$W^*_i = \frac{\theta}{4} TR + 2^{(2-1)(1-a)} (2-a) \left( \frac{a}{\theta} \right)^{x(1-a)} - 2^{(2-1)(1-a)} b^{1/(1-b)} \theta^{b(1-1)}$$

(10.1)

$$W^*_z = \frac{\theta}{4} TR + 2^{(2-1)(1-b)} (2-b) \left( \frac{b}{\theta} \right)^{b(1-b)} - 2^{(2-1)(1-a)} a^{1/(1-a)} \theta^{x(1-1)}$$

(10.2)

Eventually, note that incomes of both agents are decreasing in both degrees of returns to scale under some conditions. Verify for agent 1 that

$$\partial W^*_1 / \partial a < 0 \iff (a-2) \ln(2a/\theta) + a - 1 > 0, \partial W^*_1 / \partial b < 0 \iff (b-2) \ln(2b/\theta) + b - 1 > 0$$

and $\partial W^*_1 / \partial b < 0, \partial W^*_1 / \partial a < 0$. Then, there is a combination of $a$ and $\theta$ that makes the income of each agent decreasing in its own degree of returns to scale. In particular, the first condition states that as $\theta \to 1$ there are positive values for $a$ allowing for a negative impact of the degree of returns upon the level of income. For example if $\theta = .75$, then $\partial W^*_1 / \partial a < 0 \iff 0 < a < .24$. The intuition behind appears to be simple. In other words, when agent 1 does not retain a high degree of returns in the uncontestable sector and interprets the conflict as non-destructive, it will have fewer incentives to invest in the secure and uncontestable sector. More precisely, when agents are identical in their fighting abilities and asymmetric in their degrees of returns to scale in the uncontestable sector, a combination of the destruction parameter and the degree of returns also affect the allocation of resources. It is clear that: (a) as the degree of returns to scale in the production of ice-creams increases each agent will prefer to allocate more resources to the uncontestable sector; (b) when the conflict is perceived to be non-destructive each agent has fewer incentives to invest in the uncontestable sector. Result (a) is akin with results presented in Garfinkel and Skaperdas (2007).

**Production and Welfare**

As tools for ‘measurement’ I analyse hereafter the level of production and the total welfare. I shall consider the impact of the different variables and parameters on them. First, Using (5), (7.1) and (7.2) it is possible to compute the level of production emerging in the uncontestable sector. Then we have:

$$UY = \left( 2 \frac{a}{\theta} \right)^{x(1-a)} + \left( 2 \frac{b}{\theta} \right)^{b(1-b)}$$

(11)

First, the level of uncontestable production is unambiguously larger than zero. Eventually it is worth noting that $\partial UY / \partial a > 0 \iff \ln(2a/\theta) - a + 1 > 0$ and $\partial UY / \partial b > 0 \iff \ln(2b/\theta) - b + 1 > 0$. That is, as the conflict is perceived to be less and less destructive the degree of returns in the uncontestable sector must be
sufficiently high. Otherwise, in the presence of low returns to scale both agents would be better off by allocating resources into the contested sector. In such a case, the level of production in the uncontested sector would decrease. In other words, when the returns in the uncontested sector are extremely low the level of uncontested production would decrease. For instance, set arbitrary $\theta = .75$, in order to have a level of $UY$ increasing in $a$ and $b$ it is necessary to have $a, b > .16$. By contrast, as $\theta \to 0$ a very low degree of returns would even suffice to satisfy the positive relationship between total production in the uncontested sector and the degree of returns. Using (9.1) and (9.2) the level of production in the contested sector – namely the contested output - is given by:

$$CY = x'_1 + x'_2 = \frac{TR}{2} - 2^{(1-\theta)} \left( \frac{a}{\theta} \right)^{1/(1-a)} - 2^{(1-\theta)} \left( \frac{b}{\theta} \right)^{1/(1-b)}$$

(12)

It is trivial to say that $CY$ is increasing in both the level of resources ($\partial CY / \partial TR > 0$) and in the destruction parameter ($\partial CY / \partial \theta > 0$). At the same time it is decreasing in both $a$ and $b$, $\partial CY / \partial a < 0, \partial CY / \partial b < 0$. The higher are the returns in the uncontested sector within the bounds $(0,1)$ the lower would be the level of production in the contested sector. Total production in the economy is simply given by the sum of (9.1) and (9.2)

$$TY = CY + UY =$$

$$= \frac{TR}{2} + \theta^{1-\theta} (\theta - a) 2 a^{\theta^{1-\theta}} + \theta^{1-\theta} (\theta - b) 2 b^{\theta^{1-\theta}}$$

(13)

Also in this case it is clear that $\partial TY / \partial \theta > 0, \partial TY / \partial TR > 0$. Given the results presented above, it appears to be predictable that the degree of returns can have an ambiguous impact on the level of total production. In particular, the partial derivatives with respect to $a$ and $b$ show that:

$\partial TY / \partial a < 0 \iff (a - \theta) \ln(2a / \theta) + (a - 1)(\theta - 1) > 0$

and

$\partial TY / \partial b < 0 \iff (b - \theta) \ln(2b / \theta) + (b - 1)(\theta - 1) > 0$.

In fact, when the conflict is perceived to be more destructive both agents allocate more resources in the uncontested sector. This can decrease the level of production in the contested sector. Then, although it can appear paradoxical, this can also decrease the level of total production. This would depend upon specific combinations of $a, b$ and $\theta$. Total welfare is computed as the sum of attainable incomes:

$$TW = W'_1 + W'_2 =$$

$$= \theta \ \frac{TR}{2} + (1 - a) \left( \frac{2a}{\theta} \right)^{\theta^{1-\theta}} + (1 - b) \left( \frac{2b}{\theta} \right)^{\theta^{1-\theta}}$$

(14)
The level of total welfare is increasing in the level of resources $\partial TW / \partial TR > 0$. Note also that $\partial TW / \partial a > 0 \Leftrightarrow \ln(2a / \theta) > 0$ and $\partial TW / \partial b > 0 \Leftrightarrow \ln(2b / \theta) > 0$. Therefore, as the conflict becomes less destructive the degrees of returns in the uncontested sector must be sufficiently high. The level of total welfare is decreasing in $\theta$ if and only if the following condition is satisfied:

\[
(2a)^{(b-1)} - \theta^{(b-1)} - (2b)^{(b-1)}(b^{(b-1)}) < 0
\]

which after some manipulations can be reduced:

\[
TR(2b)^{(b-1)} - \theta^{(b-1)} < 0
\]  

(15)

Setting an arbitrary value for $TR$ it is possible to plot a parameter space $(b, \theta)$. All the points below the curves represent all the combinations of $b$ and $\theta$ that satisfy (16).

**Figure 1 – When total welfare is decreasing in $\theta$**

The plot simply shows that when the degree of returns to scale for agent 2 is sufficiently high, total welfare is decreasing in the destruction parameter even if the latter is very close to unity (namely when the conflict appears to be almost non-destructive). However, by contrast, it is clear that when the total resources endowment is sufficiently high, the level of total welfare is increasing in the destruction parameter.

The latter result suggests that in the presence of one agent sufficiently productive in the uncontested sector total welfare is no longer increasing in $\theta$. Put differently, even if the conflict is perceived to be non-destructive investing in the contested sector does not increase total welfare. This confirms the idea that the existence of conflict does not constitute a socially optimal incentive scheme. This is particularly relevant when considering that the contested sector has been assumed to be characterized by constant returns to scale, whilst the uncontested
sector has been assumed to exhibit decreasing returns to scale. To sum up it is possible to write:

**PROPOSITION 1:** when agents are identical in their fighting abilities and asymmetric in their degrees of returns to scale in the uncontested sector, even if the conflict is perceived to be non-destructive and the contested sector exhibits constant returns to scale, investing in the contested sector does not increase total welfare. In the presence of one agent sufficiently productive in the production of ice-creams, total welfare is not increasing in the destruction parameter, $\theta$.

**Redistributive Government and Rival Group**

Up to this point the analysis focused on a scenario characterized by two risk-neutral agents holding secure property rights in the production of ice-creams while contesting a joint output in a contested sector. No specific assumptions have been made about the characteristics of these agents. Hereafter, assume that agent 1 and agent 2 can be interpreted as a government and a rival group respectively. In the first scenario, a government could have been considered as a minimum government committed only to secure contracts and property rights. That is, the government is focused exclusively on enforcement of contracts. Instead, in this section, consider the existence of a redistributive government. That is, first the government can impose a tax burden on the rival subjected group. Then, assume that the government can impose a proportional tax rate on production of the subjected group in the uncontested sector. At the same time, the government can subsidize the rival group by means of redistribution of public funds. In such a case, the government behaves as a redistributive government. However, the government can be either benevolent or kleptocratic. This depends to what extent it does redistribute the tax burden to the subjected group. Both the tax burden and the redistribution of income to favour the rival group do affect the allocation of resources between butter, guns and ice-creams.

This idea is not a novelty. In particular, the tax burden imposed upon a fraction of population by ruling elites has been interpreted as a crucial factor for the emergence of revolutions. This is the basic idea surrounding some brilliant works as Grossman (1991) and Acemoglu and Robinson (2006). In the first, the author shows that a too high tax rate imposed by the ruler would increase the probability of a successful insurrection. Albeit with a different technical approach and with no distinction between butter and guns, in the latter, the authors – under different scenarios - interpret the tax rate as instrument of redistributive policies used by the governing elite in favour of the citizens so determining a revolution constraint. In fact, fearing a revolution the elite can make concessions and set a tax rate that redistribute some of the resources to the citizen. In such a framework, the revolution constraint is strongly affected by existing income inequality which can be modified through redistributive policies.

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5 I must thank an anonymous referee for this intuition.
However, given the analytical complexity, some simplifying assumptions have to be made. First, consider that both agents retain the same degree of productivity in the uncontested sector, namely \(a = b\). Then, only notation \(b\) will be used. Furthermore, assume that both agents perceive the conflict as non-destructive, namely \(\theta = 1\). Then, let \(t \in (0,1)\) denote the proportional tax rate imposed by the government on the subjected group. It is imposed on the production of ice-creams. Let also \(w \in (0,1)\) denote the proportional redistribution policy applied by government to the subjected group. For sake of simplicity no additional elements are considered (i.e. for example, there are no costs for collecting taxes). Note that \(t \geq w\). Whenever \(t = w\) the government is completely benevolent and redistributes the entire tax burden to the subjected group. Albeit absolutely unrealistic, for expository reasons, I do not exclude this possibility from the start. Moreover the redistribution is assumed to be proportional to the production of ice-creams of the subjected group. The income functions for both agents become:

\[
W^i = y^i + p(G_i, G_j)CY + ty^j - wy^j
\]  \hspace{1cm} (17.1)

\[
W^j = y^j(1-t+w) + p_j(G_i, G_j)CY
\]  \hspace{1cm} (17.2)

Hereafter for sake of simplicity, use \(q = t - w\). Of course the higher is \(q\) the less benevolent (the more kleptocratic) is the government. Both agents maximize (17.1) and (17.2) with respect to \(G_i\) and \(y_i\) with \(i = 1,2\). The second order conditions dictate the condition \(2^{b+\theta-1}TR(b-1)[b(1-q)]^{(\theta-1)} + (3-2b)(1-q)^{(\theta-1)} - 2b < -2\) for the existence of an equilibrium. As \(TR \rightarrow \infty\) the latter inequality always hold. For \(TR = 1\) condition reduces into \(2^{b+\theta-1}(b-1)[b(1-q)]^{(\theta-1)} + (3-2b)(1-q)^{(\theta-1)} - 2b < -2\). The equilibrium choices of ‘ice-creams’ are:

\[
y^*_1 = (2b)^{(\theta-1)}
\]  \hspace{1cm} (18.1)

\[
y^*_2 = -(2b(q-1))^{(\theta-1)}
\]  \hspace{1cm} (18.2)

It is clear that \(y^*_1 > y^*_2\) for \(q > 0\). It is not surprising that \(\partial y^*_2 / \partial q < 0\). That is, the tax burden depresses production in the uncontested sector for agent 2. The total production of ice-creams is given by:

\[
UY^* = (1-q)^{(\theta-1)} + 1)2b(1-q))^{(\theta-1)}
\]  \hspace{1cm} (19)

The production of ice-creams is decreasing in \(q\) and increasing in \(b\).The equilibrium choices of guns are:

\[
G^*_1 = G^*_2 = G^*_j = \frac{TR}{4} - b^{(\theta-1)}(1-q)^{(\theta-1)} + B)
\]  \hspace{1cm} (19)

where \(2^{(\theta-1)} = B\) for simplicity. The total level of guns is given by:
Of course, the total level of guns is decreasing in $b$ and increasing in both $q$ and $TR$. The level of butter is:

\[ x_1^{\nu^*} = \frac{3R - R}{4} + b^{\frac{1}{1-\omega}}(B(1-q)^{\frac{1}{\lambda - \omega}} - 3B) \]  \hfill (21.1)

\[ x_2^{\nu^*} = \frac{3R - R}{4} + b^{\frac{1}{1-\omega}}(B - 3B(1-q)^{\frac{1}{\lambda - \omega}}) \]  \hfill (21.2)

Then the total contested production of butter is:

\[ CY^{\nu^*} = \frac{TR}{2} - b^{\frac{1}{1-\omega}}\left(2B(1-q)^{\frac{1}{\lambda - \omega}} + 2B\right) \]  \hfill (22)

Total contested production is unambiguously increasing in $q$. By contrast, total contested production is decreasing in $b$ if and only if $b\ln(b(1-q)) + (1-q)^{\frac{1}{\lambda - \omega}}(b\ln(b) - b + 1) - b + 1 < 0$. That is, there are combinations of $b$ and $q$ that make the total contested production increasing in the degree of returns to scale. Figure 2 depicts a parameter space $(b,q)$ to show these combinations. Whenever $b \to 1$ and $q$ is sufficiently low the contested production is increasing in $b$.

**Figure 2 – Contested Production and Returns to Scale**

Note that $CY^{\nu^*} = UY^{\nu^*} \Leftrightarrow TR = b^{\frac{1}{1-\omega}}\left[3(2(1-q)^{\frac{1}{\lambda - \omega}} + 3 \times 2^{\frac{1}{\lambda - \omega}})\right]$. That is, there is a critical value for the entire resources endowment which – given $b$ and $q$ – allows
for equal level of production in both sectors. Eventually total production in the economy is given by:

\[
TY^{\sigma^*} = UY^{\sigma^*} + CY^{\sigma^*} = \frac{TR}{2} + b^{\sigma(1-\theta)}\left[2^{\sigma(1-\theta)}(1-q)^{\sigma(1-\theta)} + 2^{\sigma(1-\theta)}\right]
\]  

(23)

Total production is increasing in \(b\) and it is unambiguously decreasing in \(q\). The latter states that a higher tax burden leads to a lower level of production. Put differently, the more kleptocratic is the government the lower is the level of total production. Eventually final incomes of both agents are given by:

\[
W_i = \frac{TR}{4} + b^{\sigma(1-\theta)}(1-q)^{\sigma(1-\theta)}\left[B(2-b)(1-q)^{\sigma(1-\theta)} - B(q-1)(4t-1) + 2w\right]
\]  

(24.1)

\[
W_i = \frac{TR}{4} - b^{\sigma(1-\theta)}\left[B(1-q)^{\sigma(1-\theta)}(b-2) + bB\right]
\]  

(24.2)

The total welfare is the sum of (24.1) and (24.2):

\[
TW^{\sigma^*} = \frac{TR}{2} + b^{\sigma(1-\theta)}\left[(1-b)(1-q)^{\sigma(1-\theta)} + b(1-2t)(q-1) - t + 1\right](1-q)^{\sigma(1-\theta)}
\]  

(25)

Total welfare is decreasing in \(q\) and increasing in \(TR\).

**Comparative Statics**

In this brief section, a comparison between the two scenarios is presented. By means of a traditional comparative statics, I am comparing the results of the basic model analysed in the first section with those of the latter model involving the existence of a redistributive government. In particular, I will define a scenario as more or less “peaceful” by looking at the level of unproductive guns chosen by both parties. The greater the level of guns the less peaceful is that scenario considered. Given the simplifying assumptions applied in the governmental scenario (\(\theta = 1\) and \(a = b\)), equations (8), (13) and (15) will be reformulated. First, using (8) with \(\theta = 1\) and \(a = b\) the level of guns in the minimum government scenario becomes:

\[
TG = \frac{TR}{2} - (2b)^{\sigma(1-\theta)}
\]  

(26)

Then comparing (26) and (20) it is possible to verify that the level of guns in the first scenario is unambiguously lower than the level of guns chosen in the presence of a redistributive government (\(TG^* < TG^{\sigma^*}\)). Put differently, it could be stated that the minimum government scenario is more peaceful. Reformulating equation (13) with \(\theta = 1\) and \(a = b\), the level of total production in the first scenario becomes:-
$TY = \frac{TR}{2} + 2^{1/(1-b)} b^{1/(1-b)} (1 - b)$ \hfill (27)

Comparing (27) and (23) it is possible to say that $TY' > TY^{\theta^*} \Leftrightarrow (1-q)^{\theta^*} (3b - 2) + b < 0$. In figure 3 all the points on the left of the curve denote the combinations of $b$ and $q$ that make total production higher in the first scenario. Note that whenever $b$ sufficiently high, total production is is unambiguously higher in the presence of a redistributive government. This also suggests that the positive impact of a superior productivity offsets the negative impact of tax burden even in the absence of redistribution, namely when $q$ is very close to unity and the government can be defined kleptocratic.

**FIGURE 3 – COMPARATIVE STATICS OF TOTAL PRODUCTION**

Total welfare is given by:

$TW^{\theta^*} = \frac{TR}{2} + b^{k(1-b)} \left[ (1-b)(1-q)^{\theta^*} + b(1-2t)(q-1) - t + t \right] 2(1-q)^{\theta^*} \hfill (28)$

Whereas in the first scenario total welfare is given by (14) with $\theta = 1$ and $a = b$, and then by:

$TW = \frac{TR}{2} + 2^{1/(1-b)} b^{1/(1-b)} (1 - b)$ \hfill (29)

Hence, using (28) and (29) it is possible to write that $TW^{\theta^*} > TW$ if and only if:

$(1-b)(1-q)^{\theta^*} + b(q-1)(2t-1) < 1 - t \hfill (30)$
That is, there are combinations of $b, q$ and $t$ that make total welfare higher in the presence of a redistributive government. In particular, for sake of simplicity, consider some arbitrary values for $t$ in order to highlight the combination of $b$ and $q$ allowing for $TW^{\ast'} > TW$. Figure 4 depicts a parameter space $(b, q)$ to show these combinations for different arbitrary values of $t$.

**FIGURE 4 - TOTAL WELFARE, TAX BURDEN AND REDISTRIBUTION**

It is clear that a superior productivity ($b \to 1$) can increase total welfare even under the existence of a redistributive government. Instead, as $b \to 0$ inequality (30) does not hold. Put differently, whenever the degree of returns to scale is low, total welfare would be higher in the presence of a minimum government with no taxation and no redistribution. By contrast, whenever $b$ is sufficiently high there are combinations of tax burden and redistribution that allow for higher welfare under the existence of a redistributive government. In particular, it is clear that the government rent must be sufficiently low to allow for higher welfare.

In general, it appears that a scenario characterized by the existence of minimum government could be considered desirable when the degree of returns is low. Results show that it appears to be more peaceful (i.e. fewer guns), leading to both higher production and welfare. By contrast, whenever the degree of returns is sufficiently high results are ambiguous. On one hand, the existence of a redistributive government leads unambiguously to a higher level of guns that make it less ‘peaceful’. On the other hand, production and welfare can be higher in the presence of a government which collects taxes and subsidizes production of ice-creams. Therefore, even in the presence of a tax burden a proportional subsidy can boost the level of production. In particular, this appears to occur when the degree of returns is sufficiently high. Note also that with no redistribution ($w = 0$) to have $TW^{\ast'} > TW$ the tax burden must be extremely low and the degree to returns must be sufficiently high. In particular, with $w = 0$, inequality (30) reduces into $(1-b)(1-t)^{1+(t-1)} + b(2t^2 - 3t + 1) < 1 - t$. To sum up it is possible to write the following proposition:
PROPOSITION 3: when the agents are identical in both their fighting abilities and in their degrees of returns to scale in the uncontested sector then (a) in the presence of a redistributive government imposing a tax burden over a subjected group, the total level of guns is larger than in an scenario characterized by the existence of a minimum government; (b) total production is higher in the first scenario whenever both agents are low-productivity agents; (c) whenever the degree of returns is sufficiently high, total production is higher in the presence of a redistributive government; (d) whenever the degree of returns to scale is low, total welfare is higher in the minimum government scenario. By contrast, whenever it is sufficiently high there are combinations of tax burden and redistribution that allow for higher welfare under the existence of a redistributive government.

Discussion and Conclusion

This paper was an attempt to examine the conflictual interaction between two risk-neutral agents that can allocate their own resources both into a contested sector and an uncontested sector. The main results I would claim for this preliminary work is that the level of productivity in the uncontested sector can be a powerful factor inducing a higher allocation of resources in ordinary entrepreneurial activity. It is shown that the higher are the returns in the uncontested sector the lower would be the level of production in the contested sector. It is also shown that even if the conflict is perceived to be non-destructive and the contested sector exhibits constant returns to scale, investing in the contested sector does not increase total welfare. Hence, in general terms, the results of the paper recall the famous discussion posed by Baumol (1990) that suggested how entrepreneurs allocate their resources depending on the relative returns of productive and unproductive activities. The analysis confirms how the allocation of resources is significantly affected by the degrees of returns. Briefly, a sufficiently high productivity in the uncontested sector does divert resources from the contested sector to the uncontested sector increasing the opportunity cost of a bloody conflict. In other words, increased entrepreneurship can also contribute to crowd out bloody rent-seeking in contested sectors. This holds even if it is assumed that the contested sector exhibit greater returns than the uncontested sector. This partly contrasts with the argument expounded in Tornell and Lane (1999) that analyses an economy with an efficient formal sector and a less efficient informal sector. The authors show that a productivity improvement in the efficient sector does not lead to an increase in welfare when there are powerful groups demanding for discretionary redistribution. By contrast, when groups are powerless or when there recognized barriers to redistribution a productivity improvement can raise welfare. That is, the redistribution of rents between groups may outweigh the direct effect of increased productivity.

The emphasis on the impact of a superior productivity marks a difference with the argument developed in Baland and Francois (2000) where the authors emphasize that the initial equilibrium is the most important factor shaping the distribution of income between rent-seekers and entrepreneurs. In particular,
whenever an economy is characterized by a ‘full entrepreneurship equilibrium’ (that is, there are entrepreneurs in all sectors) a resource boom raises returns to entrepreneurship relative to rent-seeking. Whenever entrepreneurship does not dominate rent-seeking in the initial scenario, an exogenous resources boom lowers the returns to entrepreneurship relative to rent-seeking. Such emphasis upon the resources endowment is also in Torvik (2002) that shows how an increased amount of natural resources decreases total income and welfare. The driving assumption is that with rent seeking more profitable than modern production, entrepreneurs move into rent seeking.

Therefore, enhancing productivity in the uncontested sectors should be a desirable economic policy. However, this still represents an open question. Modelling explicitly a redistributive government and a rival subjected group leads to ambiguous results. The government collects taxes from the rival group and redistributes a fraction of tax burden through a proportional subsidy to its uncontested production. The government could be either benevolent or predatory. This affects significantly the allocation of resources. In fact, in the presence of a redistributive government investments in unproductive activities labelled as guns are larger than in a characterized by the existence of a minimum government. Hence, the latter scenario seems to be more ‘peaceful’. Whatever the degree of returns to scale, this result unambiguously holds. This seems to recall the results expounded in Bates et al. (2002) whereas the authors maintain that violence albeit intrinsically unproductive and destructive can be organized and rendered a source of welfare. However, also in this case it is clear that the degree of returns to scale has a significant impact of total production and total welfare. Total production is higher in the minimum government scenario whenever both agents are low-productivity agents. By contrast, whenever the degree of returns is sufficiently high, total production in this scenario is lower. Eventually, whenever the degree of returns to scale is low, total welfare is higher in the presence of a minimum government. If the degree of productivity is sufficiently high there are combinations of tax burden and redistribution that allow for higher welfare under the existence of a redistributive government. The latter result is a crucial point and needs further investigation.

Consider a dynamic framework. It is commonly recognized that equilibria based upon deterrence exhibit an intrinsic instability in the long run (Boulding, 1963), Greif (2007), confirms this idea explaining the self-undermining equilibrium established in medieval Genoa between rival clans. Such equilibrium was characterized by mutual deterrence between clans which continuously increased their military strength. In the long run this equilibrium became unstable leading Genoa to social unrest and civil war. Therefore, extending this model in a multiperiod framework could help to explain whether or not and under which conditions the diversion of resources from the contested sector to the uncontested sector could also lower the investments in unproductive guns in the long run.
REFERENCES


