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GENERAL EQUILIBRIUM IMPLICATIONS OF FISCAL POLICY WITH TAX EVASION: A MACROECONOMICS PERSPECTIVE

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General equilibrium implications of fiscal policy with tax evasion: a macroeconomics perspective*

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Abstract

This paper studies equilibrium effects of fiscal policy within a dynamic general equilibrium model where tax evasion and underground activities are explicitly incorporated. There are three main results. **(i)** The underground sector mitigates the distortionary impact of fiscal policies, while lessening the drop (rise) of aggregate production after contractionary (expansionary) tax shifts. In this respect, tax evasion and the informal economy offer a channel for insuring income and consumption from distortions generated by fiscal policy. **(ii)** Tax evasion and underground economy can completely reverse the theoretical predictions of the standard neoclassical growth model and rationalize expansionary responses to contractionary fiscal policies. **(iii)** A dynamic general equilibrium with tax evasion gives a rational justification for a variant of the Laffer curve.

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1 Introduction

This paper studies the macroeconomic effects of fiscal policy within a dynamic general equilibrium model in which tax evasion and underground activities are explicitly incorporated.

The macroeconomic literature on the "equilibrium approach to fiscal policy" studies effects of fiscal policy within neoclassical growth models.[1] We are not aware of any contribution, however, that evaluates the macroeconomic effect of fiscal policy explicitly incorporating tax evasion and underground activities.[2]

This might be an important part of the story since underground activities and tax evasion are a fact in many countries; for example, Schneider and Enste (2000) estimates suggest that the average size of the underground sector (as a percentage of total GDP) over 1996-97 account for 39 percent in developing countries, for 23 percent in transition countries, and for about 17 percent in OECD countries. Avoiding tax payments is the very reason for the existence of tax evasion, and this may have an important impact on the effectiveness of fiscal policy to reach the desired objectives.

To investigate relationships between underground economy, taxation and public expenditure, we use a dynamic general equilibrium model in which there are three agents: firms, households, and government.[3] In addition there are two sectors: the regular and the underground sectors. Firms and households are subject to distortionary taxation, but they can use the underground sector to evade taxes, by reallocating labor services into it. Government faces tax evasion originating from the underground sector, and coordinates strategy to address abusive tax evasion schemes. Public expenditure is allocated to the purchase the final consumption goods. Our analysis focuses on the stationary equilibrium of the model.

Here is an overview of our results. Tax evasion and the underground economy mitigate the

[1] Aschauer (1988), and Baxter and King (1993) are seminal contributions sharing an emphasis on the supply-side response of labor and capital to shifts in government demand and tax rates. Recent related contributions are: Braun (1994), McGrattan (1994), Mountford and Uhlig (2002), Burnside Eichenbaum and Fisher (2003), and Fiorito and Kollintzas (2004).

[2] McGrattan, Rogerson and Wright (1994) study fiscal policy effects in a dynamic general equilibrium model for the U.S. economy augmented with a household production sector. The model reveals the significant influence of household production in its affection on official variables. It generates different predictions for the effects of tax changes than similar models without household production. It is important to stress that an underground sector significantly differs from a household production sector, as Appendix C discusses in more details.

[3] None of the previous contributions focus on optimal fiscal policy; neither does our model, to allow a consistent comparison with this literature. For quantitative implications of optimal fiscal policy within dynamic general equilibrium models, see, for instance, Chari, Christiano and Kehoe (1995), or Cooley (1993).

distortionary impact of fiscal policies, while lessening the drop (rise) of aggregate production after restrictive (expansionary) tax changes. Tax evasion and the informal economy offer a channel for self-insuring income and consumption patterns from distortions generated by fiscal policy.[4] In particular, the elasticities of aggregate GDP to an increase (cut) in income and/or corporate tax rates are negative, but very close to zero under tax evasion, while are negative (positive) without. The negative sign of the elasticities without tax evasion is consistent with the predictions of the neoclassical growth model; on the other hand, the almost zero elasticities under tax evasion are perfectly consistent with consumption and income smoothing done through the underground sector. Finally, to have an empirical feeling for the relative magnitudes we undertake a measurement exercise estimating the empirical elasticities of aggregate GDP to a one percent increase in income or corporate tax rates. Quite interestingly, these figures are quantitatively and qualitatively consistent with the model's theoretical predictions.

In this context, tax evasion can completely reverse the theoretical predictions of the standard neoclassical growth model, under proper conditions, which are formally derived in the sequel. Tax evasion can in fact rationalize expansionary responses of an economy to contractionary fiscal policies (i.e. an increase in tax rates); notice that these effects would not be possible in a standard dynamic general equilibrium model without tax evasion. If the tax rate goes beyond a certain threshold (which is precisely identified in the paper), the additional increase in tax burden motivates households and firms to pursue abusive tax evasion schemes to avoid the excessive tax payments; operationally, firms and households reallocates labor services from the regular sector toward the underground sector, because the latter is not subject to taxation. This additional income (the tax wedge) would then be used for additional consumption and investment, pushing the economy into an expansion.[5]

An additional finding of our paper is that a dynamic general equilibrium model with tax evasion gives a rational justification for a variant of a Laffer curve.[6] It is here shown that a Laffer curve under tax evasion is *almost always* below the one computed for a 100 percent regular economy, as long

[4]We could think, for example, that the government chooses in fact the statutory tax rates, while effective tax rates are endogenously chosen by households and firms relying on the additional dimension represented by tax evasion.

[5]It is important to underline that we would observe an increase of regular GDP and of government revenues collected from the regular economy; the role of the underground sector is here as the spark that ignites the mechanism.

[6]The Laffer curve is named after Art Laffer who suggested that as taxes are increased from fairly low levels, tax revenue received by the government would also increase. However, there would come a point where people would not regard it as worth working so hard. This lack of incentives would lead to a fall in income and therefore a fall in tax revenue. The logical end point is that with tax rates at 100 percent where no one would work and so tax revenue would become zero. More details below.

as tax rates do not exceed the previously mentioned threshold. In this case, however, government revenues would be driven up by the income increase triggered by the underground sector's expansion. A Laffer curve under tax evasion is therefore characterized by an upward sloping tail, for tax rates higher than the critical threshold.

The paper proceeds as follows. Section 2 presents selected stylized facts, and Section 3 details the model. Section 4 analyzes the equilibrium effects of fiscal policies under tax evasion, and Section 5 concludes. Proofs and derivations are sketched in the Appendix.

2 Stylized Facts: Underground Economy and Tax Evasion

We present here data for the Italian economy because it possesses a large underground sector. This analysis, however, is addressed to many European countries and to the United States as well.[7] **Figure 1** below presents estimates for size of the underground economy, and for tax evasion. All series are reported as a percentage of aggregate GDP.

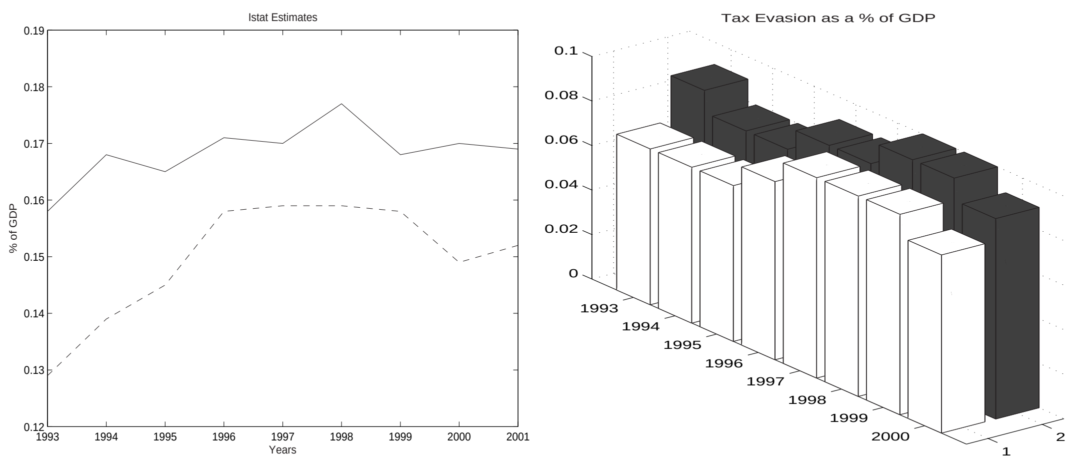

Figure 1: **Underground Economy and Tax Evasion for the Italian Economy**. Left Panel: Underground Economy as a percentage of GDP; the solid (dashed) line represents the highest (lowest) estimate; data are provided by Italy's National Statistical Institute (ISTAT) for the sample 1993:2000. Right Panel: Tax Evasion as a Percentage of GDP; the darker (white) series represents the (lowest) highest estimate; Source: Authors' calculations over the sample 1993:2000.

[7]The average size of underground activities ranges between five percent of the United States GNP (in the Seventies) and 9 percent of the United States GDP (in the Eighties, early Nintes). See Tanzi (1980), Schneider and Enste (2000), Paglin (1994). Even though these figures are below the OECD countries average (17 percent), they still represent a significant amount of resources absconded from tax collection.

The size of underground economy ranges between 13 and 18 per cent of the GDP.[8] Given the difficulty to obtain official time series statistics for tax evasion, we attempt a conservative estimate to give an idea of the figures we are talking about. Conservatively assuming that the effective tax rate for the whole economy is the minimum of the effective income (τ_Y) and firm tax rates (τ_F), we compute two approximate measures for tax evasion as $TaxEv_{\min} = (\min(\tau_F, \tau_Y)) \times u_t^{\min} \times GDP_t$ and $TaxEv_{\max} = (\min(\tau_F, \tau_Y)) \times u_t^{\max} \times GDP_t$, where $u_t^{\min}$ and $u_t^{\max}$ denote lowest and highest official estimates for underground economy share as a percentage of the GDP. The right panel of Figure 1 shows that tax evasion accounts for at least 5 percent of the GDP. This is a quite big figure, and an analogous exercise for other European countries and the United States generates qualitatively comparable figures. If governments were effectively able to recollect unpaid taxes, this would generate, on a yearly basis, a significant increase of government revenues.

3 The Model's Structure

To investigate relationships among underground economy, taxation and public expenditure we use a dynamic general equilibrium model in discrete time. There are three agents in the model: the firms, the households, and the government. In addition there are two sectors: the regular and the underground sectors. Firms and households are subject to distortionary taxation, but they can use the underground sector to evade taxes, by reallocating labor services across sectors.

The firms produce a homogeneous good by combining three production factors: physical capital, regular and irregular labor services. The latter represents the channel through which tax evasion is undertaken.

The households choose consumption, investment, and labor services on each date and in each sector (regular and underground) to maximize the expected discounted value of utility, subject to a sequence of budget constraints, a proportional tax rate on "regular income", and the law of motion for capital stock.[9] The tax system is as parsimonious as possible: the tax base is easily identified and the tax rates (either income or corporate) are constant and proportionally related to the tax base.

[8] See Baldassarini and Pascarella (2003).

[9] The "regular income" includes income flows generated in the regular sector, including also returns to capital stock. These are declared to the Internal Revenues Services; on the contrary, income flow generated from the underground sector is not included into the tax-base.

Finally, government levies proportional taxes on revenues and incomes, and balances its budget (in expected terms) for each period. In this context, government faces tax evasion originating from the underground sector, and coordinates strategy to address abusive trust schemes. Violations of the Internal Revenue Service Codes may result in civil penalties and/or criminal prosecution, which we model as a surcharge factor over customary tax rates (more details below). Public expenditure is allocated to the purchase of the final consumption good.

3.1 Firms

3.1.1 Production Technologies

Suppose that there exists a continuum of firms, uniformly distributed over the unit interval. Each firm $i \in [0,1]$ produces final output by using two different technologies, one associated with the regular sector $y^i_{M,t}$, and the other with the underground sector $y^i_{U,t}$.

$$y^i_{M,t} = \left(k^i_t\right)^{\alpha} \left(n^i_{M,t}\right)^{1-\alpha} \quad \text{and} \quad y^i_{U,t} = \left(n^i_{U,t}\right)^{1-\sigma}, \quad \alpha, \sigma \in (0,1), \tag{1}$$

where the regular output, $y^i_{M,t}$, is the result of private capital k^i_t , and regular labor, $n^i_{M,t}$ applied to a Cobb-Douglas technology. The underground output, $y^i_{U,t}$, is produced with a production function which uses only underground labor $n^i_{U,t}$ and it displays decreasing returns to scale.[10] The assumption of decreasing returns to scale in the underground labor is based on the existence of un-modeled fixed factors, such as managerial ability to abscond the corresponding tax base from the taxation, or land.

3.1.2 Revenues and Tax Evasion

Denote a price vector for this economy as $\langle \tilde{q}_{M,t}, \tilde{q}_{U,t}, \tilde{w}_{M,t}, \tilde{w}_{U,t}, \tilde{r}_t \rangle$, where $\tilde{q}_{M,t}, \tilde{q}_{U,t}$ represent, respectively, prices for the regularly-produced and the underground-produced commodity, $\tilde{w}_{M,t}, \tilde{w}_{U,t}$ denote labor wages, and $\tilde{r}_t$ is returns to capital. Since we assume that there exists a homogenous consumption good, then the two prices are equal, i.e. $\tilde{q}_{M,t} = \tilde{q}_{U,t} \equiv \tilde{q}_t$. Normalizing the commodity price $\tilde{q}_t$ to unity, the normalized price vector supporting the equilibrium equals $\langle 1, w_{M,t}, w_{U,t}, r_t \rangle$, where $w_{M,t}, w_{U,t}$ and r_t denote equilibrium real wage rates and the real return on capital (see below).

[10]The model can be relabeled by interpreting the regular sector as the manufacturing sector, the underground economy as the service sector, and introducing a relative price for the different commodities. “Manufacturing” uses labor and capital and “services” use just labor with the decreasing returns to scale technology. The analyzed fiscal policy shocks can be reinterpreted as changes in these different tax rates.

Since $\tilde{q}_t = 1$ holds in the equilibrium, we can impose it along the solution. Aggregate output equals therefore the sum of regular and underground produced output: $y_t^i = y_{M,t}^i + y_{U,t}^i$.

Regularly-produced revenues, $\mathcal{R}_{M,t} = (1-\tau_F)y_{M,t}^i$, are taxed at the rate τ_F, $\tau_F \in (0,1)$. Firms do not pay taxes on underground produced revenues, $\mathcal{R}_{U,t} = y_{U,t}^i$. Firms, however, may be discovered evading, with probability $p \in (0,1)$, and forced to pay the tax rate, τ_F, increased by a surcharge factor, $s > 1$, applied to the standard tax rate.[11] Condition 1 below assumes that the effective tax rate paid when firms are detected is higher than the statutory one ($\tau_F s > \tau_F \Rightarrow s > 1$), but it also suggests that the expected tax payment when evading should be less than the statutory one ($\tau_F sp < \tau_F \Rightarrow sp < 1$); otherwise there would not be tax evasion.

Condition 1 (Penalty and Detection Probability) *$s > 1; sp < 1 - p$.*

Finally, the chart below summarizes firm revenues' structure in the two states:

$\mathcal{R}_t^i$	$\rightarrow$	**D**etected ($\sim p$)	$\mathcal{R}_{D,t}^i = (1-\tau_F)y_{M,t}^i + (1-s\tau_F)y_{U,t}^i$
	$\searrow$		
		Not **D**etected $\sim (1-p)$	$\mathcal{R}_{ND,t}^i = (1-\tau_F)y_{M,t}^i + y_{U,t}^i$

Total expected revenues are thus:

$$\mathbb{E}_t \mathcal{R}_t^i = (1-\tau_F)y_{M,t}^i + (1-ps\tau_F)y_{U,t}^i. \tag{2}$$

Condition 2 (No Bankruptcy) *$(1 - ps\tau_F) > 0$ and $ps \leq (1-p)$.*

Notice that a firm cannot go bankrupt, since $1 - ps\tau_F$ is positive in equilibrium. According to the Italian Tax Law s is calibrated equal to 1.3, and the equilibrium value of τ_F equals 0.4155 (more details to come). This ensures that $1 - s\tau_F > 0$. The second part of the condition states that that the expected surcharge (ps) should be less than $(1-p)$ otherwise the expected returns to a unit of evaded production, $(1-p)\,\tau_F - p\tau_F s$, would be negative, and the firm would have no convenience to operate in the underground sector. The cost structure is presented below.

[11] This quantity is chosen by relying on the Italian Tax Law, because we calibrate the model for this economy. More details are presented below.

3.1.3 Costs' Structure and Profit Maximization

Following Prescott and Mehra (1980), we assume that each firm solves a myopic profit maximization problem, on a period-by-period basis, subject to a technological constraint, and to the possibility that it may be discovered producing in the unofficial economy, convicted of tax evasion and subject to a penalty surcharge. We assume optimizing and price taking behavior on the part of all agents, households and firms. Specifically, firms maximize profits on a period-by-period basis.

The cost of renting capital equals its marginal productivity r_t, net of capital depreciation, Ω. The cost of labor is represented by the wage paid for hours worked.[12] At each date t, firm i maximizes period expected profits π_t^i:

$$\begin{aligned}
\max_{\{n_{M,t}^i, n_{U,t}^i, k_t^i\}} \pi_t^i &= \mathbb{E}_t(\mathcal{R}_t^i - w_{M,t}n_{M,t}^i - w_{U,t}n_{U,t}^i - r_t k_t^i) \qquad (3)\\
\text{subject to} &: \; y_{M,t}^i = (k_t^i)^\alpha (n_{M,t}^i)^{1-\alpha},\; y_{U,t}^i = \left(n_{U,t}^i\right)^{1-\sigma}\\
&: \; \mathbb{E}_t\mathcal{R}_t^i = (1-\tau_F)y_{M,t}^i + (1-ps\tau_F)y_{U,t}^i\\
&: \; n_{M,t}^i > 0, n_{U,t}^i > 0, k_t^i > 0.
\end{aligned}$$

– *Corporate Efficiency Conditions.*

In a competitive equilibrium factors are marginally priced, as the following first order conditions suggest:

$$\left\{ \begin{array}{c}
w_{M,t} = (1-\tau_F)(1-\alpha)\left(k_t^i\right)^\alpha (n_{M,t}^i)^{-\alpha}\\
w_{U,t} = (1-ps\tau_F)(1-\sigma)(n_{U,t}^i)^{-\sigma}\\
R_t = (1-\tau_F)\,\alpha\left(k_t^i\right)^{\alpha-1}(n_{M,t}^i)^{1-\alpha}.
\end{array} \right\} \qquad (4)$$

The firm's equilibrium behavior is characterized by the previous necessary and sufficient conditions. The decreasing returns to scale in the underground sector generates excess profits $\pi_t^i > 0$, which are generated by the underground labor supply. In equilibrium, profits equal:

[12] A more general structure would account for labor costs, too (e.g. social security contributions). This would mean that a worker's cost is augmented by social security contributions only for the regular working time, while there is no tax wedge on the remaining *hidden hours*. This model, however, abstracts from this additional tax rate and leaves its analysis to future investigations.

$$\pi_t^i = (1 - ps\tau_F)\left(n_{U,t}^i\right)^{1-\sigma}\frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma, \tag{5}$$

and are uniformly distributed among all households.

3.2 Households and preferences

The representative household, indexed with $\gamma \in [0,1]$, has preferences over consumption and labor services. For most of our analysis we specialize momentary utility to have the form:[13]

$$\mathcal{U}_t^\gamma = \log\left(c_t^\gamma + \phi c_{G,t}\right) - B_M(n_{M,t}^\gamma + n_{U,t}^\gamma) - B_U n_{U,t}^\gamma, \quad B_M, B_U \geq 0, \tag{6}$$

where c_t^γ denotes the private consumption profile of household γ, $n_{M,t}^\gamma$ her regular labor services supply, and $n_{U,t}^\gamma$ her underground labor supply; $c_{G,t}$ denotes per-capita government purchases of the homogeneous good produced in the economy, and ϕ is a parameter representing the degree of substitutability between government and private consumption flows.[14] When $\phi = 1$ ($\phi < 1$) private and public consumption goods are perfect complement (substitute). Households would react to a one-unit increase in public consumption by lowering private consumption by one unit. Given that $c_t^\gamma + \phi c_{G,t} > 0$, this impose a further restriction on $\phi > -\frac{c_t^\gamma}{c_{G,t}}$. When $\phi = 0$, $c_{G,t}$ does not affect the households' utility.[15] It is assumed that households take $c_{G,t}$ as given, and that there is no congestion effect.[16]

The quantity $B_M\left(n_{M,t}^\gamma + n_{U,t}^\gamma\right)$ represents the total disutility of working, while the last term, $B_U n_{U,t}^\gamma$ reflects an idiosyncratic cost of working in the underground sector. Specifically, this cost

[13] Our utility function departs from Baxter and King (1993)'s formulation. Their specification is separable among private consumption C_t, leisure L_t, and government expenditure (basic consumption G_t^B and public capital stock K_t^G); using their notation it reads: $u\left(C_t, L_t, G_t^B, K_t^G\right) = u\left(C_t, L_t\right) + \Gamma\left(G_t^B, K_t^G\right)$, where $\Gamma_1, \Gamma_2 > 0$. This preference specification is meant to capture a government spending that does not directly affect private consumption, i.e. military spending.

[14] Also notice that since households are uniformly distributed over the unit interval and there is no population growth, per-capita government spending coincides with the corresponding aggregate quantity. We assume that the households take this externality as given.

[15] The role of government expenditure into our utility function is slightly different from the customary one. Precisely, c_G denotes per capital expected government expenditure, because it incorporates expected revenues. The expectation originates from the probability of detecting a firm evading and collecting the absconded tax payments. It should be noted, moreover, that as long as an household allocates labor services to both sectors, she has always incentives to evade taxes. That happens because the value of the additional (tax-free) income is higher than the the value of government consumption added to the individual consumption flow.

[16] We treat government expenditure as a pure public good, and we abstract from congestion typically associated with public goods, in order to sharply identify the tax-evasion impact on fiscal policies. We leave the introductions of different notions of congestions to future investigations.

may be associated with the lack of any social and health insurance in the underground sector. This utility function is separable between consumption and labor and allows to study how a household allocates its labor supply between the regular and the underground sectors.

Section 3.4.1 below shows that the following condition ensures that the households would find optimal to supply labor in both sectors. It says that the idiosyncratic cost associated with working in the underground sector should not be *too large*:[17]

Condition 3 (Labor Market Equilibrium) $B_U = \frac{w_U - w_M(1-\tau_Y)}{C}$,

where C denotes aggregate consumption (i.e. the sum of private and public consumptions, taking into account a possibly different degree of substitutability), w_U and $(1-\tau_Y)w_M$ respectively represent underground and net-of-tax regular salaries (defined before). Notice that the higher the aggregate consumption, the lower is the idiosyncratic cost of working into the underground economy.[18]

In each period the representative household faces a resource constraint saying that the total use of goods for consumption and investment cannot exceed the disposable income, net of income taxes, $\tau_Y \in (0,1)$:

$$c_t^\gamma + i_t^\gamma = (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_U n_{U,t}^\gamma + \pi_t, \tag{7}$$

where $w_{M,t}$ and $w_{U,t}$ represent regular earnings and earnings from the underground sector, respectively; income generated from the underground sector $w_U n_{U,t}^\gamma$ and per-capital profits $\int_0^1 \pi_t^i di = \pi_t$ are absconded away from income taxation.[19]

Finally, investment increases the capital stock according to a customary state equation:

$$k_{t+1}^\gamma - (1-\Omega)k_t^\gamma = i_t^\gamma,$$

where Ω denotes a quarterly depreciation rate for private capital stock.

The $\gamma - th$ household's sequential problem is therefore the following:

[17] See Section 3.4.1 for a more detailed discussion and the derivation.

[18] In this respect a valuable government expenditure does not rule out the existence of an idiosyncratic cost for the underground labor.

[19] Firms and households are uniformly distributed over the unit interval. Here we implicitly assume that firms' profits are uniformly distributed over the unit mass of households. Therefore distributing the aggregate profits $\int_0^1 \pi_t^i di$ over the unit mass of households yields that per-capita profits equal $\int_0^1 \pi_t^i di = \pi_t$.

$$\max_{\{c_t^\gamma, i_t^\gamma, n_{M,t}^\gamma, n_{U,t}^\gamma\}_{t=0}^\infty} \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t^\gamma \tag{8}$$

$$\begin{aligned} s.t. \quad &: \quad c_t^\gamma + i_t^\gamma = (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_U n_{U,t}^\gamma + \pi_t \\ &: \quad i_t^\gamma = k_{t+1}^\gamma - (1-\Omega) k_t^\gamma \\ &: \quad k_{i,0}^\gamma,\ \pi_t \text{ and } c_{G,t} \text{ given, } c_t^\gamma, n_{M,t}^\gamma, n_{U,t}^\gamma > 0 \text{ and } B_U \leq \frac{w_U - w_M(1-\tau_Y)}{C}, \end{aligned}$$

where households take the government purchase of final commodities $c_{G,t}$ and corporate profits as given. Pooling together the feasibility and the capital accumulation constraint, the constraint set is $c_t^\gamma + k_{t+1}^\gamma - (1-\Omega)k_t^\gamma = (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_{U,t} n_{U,t}^\gamma + \pi_t$.

– *Private Efficiency Conditions.*

A representative household chooses how much to consume (c_t^γ), how many labor services to allocate in each sector ($n_{M,t}^\gamma$ and $n_{U,t}^\gamma$), and next period capital stock (k_{t+1}^γ). More formally, her behavior is characterized by the following necessary and sufficient first order conditions:

$$\left\{ \begin{gathered} B_M = \left(c_t^\gamma + \phi c_{G,t}\right)^{-1} (1-\tau_Y)\, w_{M,t} \\ B_M + B_U = \left(c_t^\gamma + \phi c_{G,t}\right)^{-1} w_{U,t} \\ c_t^\gamma + k_{t+1}^\gamma = (1-\Omega)k_t^\gamma + (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_{U,t} n_{U,t}^\gamma + \pi_t. \end{gathered} \right\} \tag{9}$$

Next, optimal investment choice by the representative agent depends on the following Euler Equation:

$$\left(c_t^\gamma + \phi c_{G,t}\right)^{-1} = \beta \mathbb{E}_t \left(c_{t+1}^\gamma + \phi c_{G,t+1}\right)^{-1} \left((1-\tau_Y)\, r_{t+1} + 1 - \Omega\right). \tag{10}$$

3.3 Government

The government is described as a sequence $\{\varrho_t\}_{t=0}^\infty$ of tax rates on households' income, on firms' revenues and of government spending G_t

$$\{\varrho_t\}_{t=0}^\infty = \{\tau_Y, \tau_F; G_t\}_{t=0}^\infty. \tag{11}$$

Government spending is determined endogenously in equilibrium to balance the public budget constraint.[20] Next, collected tax revenues, denoted by RV_t, read

$$RV_t = [(w_{M,t}N_{M,t} + r_tK_t)\,\tau_Y + \tau_F\,(psY_{U,t} + Y_{M,t})] = c_{G,t}, \tag{12}$$

where capitalized letters denote aggregate equilibrium quantities, which are defined as $Y_{M,t} = \int_0^1 y^i_{M,t}di$, $Y_{U,t} = \int_0^1 y^i_{U,t}di$, $N_{M,t} = \int_0^1 n^i_{M,t}di$, $K_t = \int_0^1 k^i_t di$. Income tax is collected from households over regular wage and capital revenues $(w_{M,t}N_{M,t} + r_tK_t)$, and from firms over regular $\tau_F\,(Y_{M,t})$, and underground produced revenues $\tau_F\,(Y_{U,t}ps)$. The latter quantity denotes expected tax revenues flows, because it takes into account that part of the tax-base is successfully absconded from Internal Revenues Service, with a positive probability p. Government spending is allocated to purchase of final consumption goods $c_{G,t}$ and it equals government revenues.[21]

In this model both households and firms evade taxes. Households evade income taxation producing a tax loss associated to the underground-produced income flow $\tau_Y w_{U,t}n_{U,t}$ and distributed per-capita profits π_t. Firms always try to evade an amount of taxes equal to $\tau_F y^i_{U,t}$; when a firm is detected, however it pays the additional fine $s\tau_F y^i_{U,t}$. That happens with probability p. In the other case, firms do evade, and are not detected; this event happens with probability $1-p$.[22] Hence expected corporate tax evasion is $(1-p)\,\tau_F y^i_{U,t}$.[23] Combining these quantities, total expected tax evasion reads $\mathbb{E}_t TE_t = (1-p)\,\tau_F y^i_{U,t} + \tau_Y w_{U,t}n_{U,t}$.

3.4 Competitive Equilibrium Characterization

A Competitive Allocation is a policy $\{\varrho^*_t\}^\infty_{t=0}$, an allocation $\{x^*_t\}^\infty_{t=0} = \left\{K^*_t, N^*_{M,t}, N^*_{U,t}, C^*_t\right\}^\infty_{t=0}$ and a price system $\left\{1, w^*_{M,t}, w^*_{U,t}, r^*_t\right\}^\infty_{t=0}$ such that, given the policy and the price system, the resulting allocation maximizes the representative household utility (conditions (9)) subject to:

(i) the sequence of budget constraints, (condition (7));

(ii) the price system $\left\{1, w^*_{M,t}, w^*_{U,t}, r^*_t\right\}^\infty_{t=0}$ (conditions (4));

[20] Notice that this paper does not present an "optimal taxation exercise", in the sense that public consumption or investment choice is not derived from an optimization procedure. In this respect our framework departs from Chari, Christiano and Kehoe (1995), while it follows McGrattan (1994).

[21] The model abstracts from debt accumulation since the government balances its budget on a period by period basis, as equation (12) suggests. Consumption expenditure, furthermore, is fully revenue-financed, and it is implicitly assumed that the government does not issue bonds.

[22] It is also assumed that the proportion of firms evading taxes does not change from period to period.

[23] Firms and households differ along the following dimension: households cannot be discovered, while firms are subject to the probability of being discovered and then fined.

(iii) the government budget constraint being satisfied on average, (condition (12));

(iv) market clearing conditions holding for each market, and the following aggregate constraint for the economy being satisfied:

$$C_t + I_t + G_t = Y_t,$$

where G_t denotes aggregate government spending, and aggregate consumption, investment, and output are defined as $C_t = \int_0^1 c_t^\gamma d\gamma$, $I_t = \int_0^1 i_t^\gamma d\gamma$ and $Y_t = \int_0^1 y_t^i di$, respectively.

The following Proposition 1 shows that the model has a unique stationary state for capital stock, a unique value for equilibrium regular and underground labor services.

Proposition 1 *There exists a unique stationary capital stock* $\overline{K} > 0$*, and a unique stationary equilibrium for regular labor* $\overline{N}_M > 0$*, and underground labor* $\overline{N}_U$ *such that:*

$$\begin{aligned}
\overline{N}_M &= \frac{\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\left(\bar{\Psi}\right)^\alpha - \lambda_U N_U^{1-\sigma}\left[1-(1-\phi)\,ps\tau_F\right]}{\left[(1-\tau_Y)(1-\tau_F)+\phi\left((1-\tau_F)\tau_Y+\tau_F\right)\right]\lambda_M\left(\bar{\Psi}\right)^\alpha - \Omega\bar{\Psi}} \\
\overline{N}_U &= \left[\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\right)^{-\frac{\alpha}{\sigma(1-\alpha)}}\left(\frac{B_M}{B_U+B_M}\frac{\lambda_U}{\lambda_M}\right)^{\frac{1}{\sigma}}\left(\frac{1-\sigma}{1-\alpha}\right)^{\frac{1}{\sigma}}\right]\times \\
&\quad \times\left[\left(\frac{1}{1-\tau_Y}\right)^{\frac{1}{\sigma(1-\alpha)}}\left(\frac{1}{1-\tau_F}\right)^{\frac{1}{\sigma(1-\alpha)}}\left(1-ps\tau_F\right)^{\frac{1}{\sigma}}\right] \\
\overline{K} &= \left(\frac{\alpha\,(1-\tau_Y)(1-\tau_F)}{\beta^{-1}-1+\Omega}\right)^{\frac{1}{1-\alpha}}\overline{N}_M
\end{aligned}$$

where $\frac{K}{N_M} = \left(\alpha\lambda_M\frac{(1-\tau_Y)(1-\tau_F)}{\beta^{-1}-1+\Omega}\right)^{\frac{1}{1-\alpha}} = \bar{\Psi}.$

Proof. See Appendix A. ∎

Once we have equilibrium values for the stationary capital stock and labor inputs, the remaining equilibrium quantities (consumption, regular output, underground output, aggregate output, investments) are derived from the budget constraint, the production functions and the capital accumulation constraint, all evaluated at the stationary state.

As the quantities in Proposition 1 suggest, the unique stationary state is characterized by a positive equilibrium level for regular and underground labor services. The only economically meaningful scenario in which $\overline{N}_U = 0$ is when $1 = ps\tau_F$ or when $B_U \to \infty$; these case are, however, ruled out by the "no-bankruptcy condition" (see Condition 2) and by the "labor market equilibrium condition" (Condition 3). The next section describes in more details the labor market equilibrium.

3.4.1 Labor Market Equilibrium

Consider, first, the supply side of the underground labor market. A utility maximizing household would allocate labor services to a true underground sector (that is meant to evade taxes) if its additional cost (B_U) is less than (equal at the margin) that produced by distortionary taxation. Therefore a maximizing household would compare the net-of-tax wage ($w_M (1 - \tau_Y) = CB_M$) with the net-of-idiosyncratic cost wage ($w_U - CB_U = CB_M$);[24] at the margin these two quantities should be equal, that is $w_M (1 - \tau_Y) = w_M - CB_U$, which implies:

$$w_M (1 - \tau_Y) = w_U - CB_U \Rightarrow B_U = \frac{w_U - w_M (1 - \tau_Y)}{C}, \tag{13}$$

where C denotes aggregate consumption, and salaries w_M and w_U are taken as given by the households. This is Condition 3 in Section 3. In other words, the idiosyncratic additional cost associated to underground labor services exploits the opportunity cost related to evading the tax wedge $w_U \tau_Y$. Now, as long as condition (13) is satisfied, a household would supply labor in both sectors. In addition, From the first order conditions for the households we obtain that $B_U = B_M \frac{w_U - w_M (1-\tau_Y)}{w_M (1-\tau_Y)}$, which implies, in turn, that optimal equilibrium labor supplies would be such to equate, at the margin, the two kinds of disutilities (the distortionary taxation and the additional riskiness of the underground labor).[25] When this condition is satisfied (as it happens in equilibrium), it follows that condition (13) holds as well, since B_M is calibrated to a sufficiently small value (see Section 4.1 below). It can be concluded that, in equilibrium, households would supply labor services in both sectors.

Consider, next, the demand side of the labor market. Profit maximizing firms equate the gross-of-tax regular wage $\frac{w_M}{1-\tau_F} = (1 - \alpha) (K)^{\alpha} (N_M)^{-\alpha}$ with gross-of-expected tax underground salary $\frac{w_U}{1-p s \tau_F} = (1 - \sigma)(N_U)^{-\sigma}$. The technology structure ensures, in addition, that a corner solution (that

[24] From a geometrical perspective in the plane employment-salary, notice that both the idiosyncratic riskiness of underground labor (the parameter B_U) and the distortionary income taxation (the quantity $(1 - \tau_Y)$) shift upward households' labor supply schedule.

[25] This implies that the households would appropriate the entire tax wedge. It would also be interesting to consider more sophisticated allocation mechanisms, i.e. a bargaining solution.

is $n^i_{M,t} = 0$ or $n^i_{U,t} = 0$) would not be an optimal solution for the firms' optimization problem, that is for the demand side of the model.[26] This argument would support a non-negative demand for each labor input.

4 Results

This section discusses how tax evasion and underground activities modify the macroeconomic consequences of permanent shifts in income and corporate tax rates. We focus our attention on the stationary state.

4.1 Parametrization

The model is parameterized for the Italian economy. The system of equations we use to compute the dynamic equilibria of the model depends on a set of nine parameters. **Four** pertain to household preferences (B_M, B_U, β, ϕ), **two** to the structural-institutional context (the probability of a firm being detected p and the surcharge factor s), and the remaining **three** parameters to technology (the capital share in the production function α, the private capital stock quarterly depreciation rate Ω, and the underground labor elasticity $1 - \sigma$).

Labor supply parameters (B_M, B_U) : the disutility parameters B^*_M and B^*_U are calibrated equal to 0.0482 and 0.0081, respectively, to match the logarithms of the average of the trend component for regular and underground employment; $\log \overline{N}_M$ and $\log \overline{N}_U$ equal to 16.44 and 14.77, respectively. Data are from the Italian Statistical Institute (ISTAT) over the sample 1992-2001.

Preference and Technology $(\alpha, \beta, \Omega, \sigma)$ are set to commonly used values in this literature (e.g. Fiorito and Kollintzas, 1994). More precisely, we set $\beta^* = 0.984$, $\Omega^* = 0.025$, and $\alpha^* = 0.36$. Concerning the returns to scale in underground production $(1 - \sigma)$, we set $\sigma^* = 0.483$.

The **degree of substitutability** between government and private consumption flows (ϕ) is calibrated following Fiorito and Kollintzas (2004). These authors, using a balanced panel with twelve country sample ranging from 1970 to 1996, estimate the relationship between public and private consumption splitting the former into two categories. The "public goods" which includes

[26] The Inada conditions,next, ensure that the productivity (and therefore the salary) of both types of labor goes to infinity when the corresponding labor service goes to zero. Precisely: $\lim_{n^i_{M,t} \to 0} w_{M,t} = \infty$ and $\lim_{n^i_{U,t} \to 0} w_{U,t} = \infty$, where $w_{M,t}$ and $w_{U,t}$ denote the regular and underground labor demand schedule, as conditions (4) state.

defense, public order and justice, and the category of "merit goods" which includes health, education and other services that could have been provided privately. This latter category is about two third of government consumption. The GMM estimates stress that merit goods complement private consumption. The long-run estimated elasticity of private consumption to government merit-goods consumption is high and ranges from 0.55 to 0.97. Fiorito and Kollintzas show that this result is quite stable; that is, signs and relative size of the parameters are robust to several measurements of the variables and equations. The government expenditure present in our model is akin to what Fiorito and Kollintzas define as merit goods; then, we set $\phi^* = 0.76$, which is the mean between the two estimated values. A sensitivity analysis, available upon request, shows that the results are qualitatively comparable for the other values in the range suggested by Fiorito and Kollintzas (2004).

The **probability of being detected** is set to $p^* = 0.03$, and the **penalty factor** is calibrated to $s^* = 1.3$, as suggested by Busato and Chiarini (2004).

Concerning the **corporate tax rate**, in Italy, corporations are subject to a progressive tax rate. A tax rate of 19 percent is applied to the share of profits that represents 7 percent of the firm's capitalization; the remaining portion is then subjected to an increased tax rate of 36 percent. We calibrate the steady state value of the corporate tax rate as the average of these two numbers, i.e. $\tau_F^* = 0.275$.

The personal **income tax** system is more complex, since Italy has five tax rates, spanning from 18.5 percent to 45.5 percent.[27] The calibration of the income tax rate may be undertaken in two ways. It may be estimated as the average tax rate, weighted by the relative share of population in each income class. It may also be estimated as the tax rate associated with the average income of the working population (adults 15-64 years old). We rely on the second procedure and since the average income equals 18,246 Euros we estimate the income tax rate at 33.5 percent.

Finally, notice that the model we use for assessing the consequences of fiscal policy along the stationary equilibria is consistent with the selected long-run statistics measured for the Italian economy. In this sense, the model could be consistently used for undertaking fiscal policy experiments.

[27] More precisely, the structure of the tax rates is the following as of 2001. For incomes less than 10,331 Euros the tax rate is 18.5 percent, for incomes between 10,331 Euros and 15,496 Euros the tax rate is 25.5 percent, for incomes between 15,496 Euros and 30,992 Euros the tax rate is 33.5 percent, for incomes between 30,992 Euros and 63,283 Euros the tax rate is 39.5 percent and, finally, for incomes above 63,283 Euros the tax rate is 45.5 percent. More details can be found at the web-sites *www.finanze.it* or *www.tesoro.it*.

Table 1: **Actual and calibrated "great ratios"**

$\left(\frac{C}{Y}\right)^* = 0.86$	$\left(\frac{C_G}{Y}\right)^* = 0.32$	$\left(\frac{I}{Y}\right)^* = 0.09$	$\left(\frac{Y_U}{Y}\right)^* = 0.14$	$\left(\frac{N_U}{N}\right)^* = 0.21$
$\widehat{\left(\frac{C}{Y}\right)} = 0.77$	$\widehat{\left(\frac{C_G}{Y}\right)} = 0.38$	$\widehat{\left(\frac{I}{Y}\right)} = 0.08$	$\widehat{\left(\frac{Y_U}{Y}\right)} = 0.16$	$\widehat{\left(\frac{N_U}{N}\right)} = 0.25$

Notes: $\frac{C}{Y}$ denotes the ratio between aggregate consumption and aggregate GDP; $\frac{I}{Y}$: ratio between net investments and aggregate GDP; Sources: National Statistical Institute (ISTAT) (www.istat.it/english); National Account Data; 1970-2004. $\frac{Y_U}{Y}$: underground production share; $\frac{N_U}{N}$: underground employment share; National Statistical Institute (ISTAT) for the sample 1993:2000.

In particular, Table 1 below presents selected "equilibrium ratios" generated from the model (starred quantities) and estimated for the Italian economy (hat quantities).

4.2 Macroeconomic effects of fiscal policy with tax evasion

This section discusses selected consequences of fiscal policies over the stationary equilibrium of the model evaluating the role of tax evasion and the underground sector. To carry out this analysis the following Propositions derive the elasticities of aggregate GDP (defined as the sum of regular and underground production) to permanent changes in income (τ_Y) and corporate tax rates (τ_F). The sequel of the paper discusses only the case of an increase in either tax rates, being the model's response to a tax cut symmetric.

4.2.1 Analytical derivation of aggregate GDP elasticities to tax rates

Proposition 2 below derives the long-run elasticities of aggregate production to a permanent shift in income tax rate.

Proposition 2 (GDP Elasticity to τ_Y) *The elasticity of aggregate GDP* $y_t = y_{M,t} + y_{U,t}$ *to a* *permanent increase in income tax rate τ_Y is denoted as ε_{Y,τ_Y} and reads:*

$$
\begin{array}{llr}
\varepsilon_{Y,\tau_Y} = & & \\
(A) & -\frac{\alpha}{1-\alpha}\frac{\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}(1-\tau_F)}{\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}(1-\tau_Y)(1-\tau_F)\right)} & (<0) \\
(B) & -\left[\frac{1}{D_1}\frac{(1-\tau_F)(1-\alpha)}{B_M}\bar{\Psi}^{\alpha}\right]+ & (<0) \\
(C) & +\left[\frac{1}{D_1}\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\alpha\left(\bar{\Psi}\right)^{\alpha-1}\frac{\partial\bar{\Psi}}{\partial\tau_Y}\right]+ & (<0) \\
(D) & -\left[\frac{1}{D_1}(1-\sigma)\lambda_U N_U^{-\sigma}\frac{\partial N_U}{\partial\tau_Y}\left[1-(1-\phi)ps\tau_F\right]\right]+ & (<0) \\
(E) & +\left[\Omega\frac{\partial\bar{\Psi}}{\partial\tau_Y}\frac{1}{D_2}\right]+ & (<0) \\
(F) & -\left[(1-\phi)(1-\tau_F)\lambda_M\frac{1}{D_2}\bar{\Psi}^{\alpha}\right]+ & (<0) \\
(G) & -\left[((1-\tau_Y)(1-\tau_F)+\phi((1-\tau_F)\tau_Y+\tau_F))\lambda_M\alpha\left(\bar{\Psi}\right)^{\alpha-1}\right]\frac{1}{D_2}\frac{\partial\bar{\Psi}}{\partial\tau_Y}+ & (>\mathbf{0}) \\
 & & \\
(J) & +\left[\frac{1-\sigma}{\sigma}\frac{1}{1-\alpha}\frac{1}{1-\tau_Y}\right] & (>\mathbf{0})
\end{array}
$$

Proof. See Appendix A. ■

Now, an increase of the income tax rate has a negative impact on aggregate production because of the direct effect of the income tax rate (A) to (F), but a positive one driven by the increase in labor input in the underground sector (J) and by returned government expenditure (G) (i.e. the so called tax exchange) Therefore the underground sector mitigates the distortionary impact of fiscal policies, while lessening the fall of aggregate production after restrictive tax changes. A tax cut has a symmetric impact, *mutatis mutandis.* In this sense, tax evasion (underground sector) offers *insurance* to income tax rate shifts. The concept of insurance is present in the sense that underground-produced income flows completely avoid distortionary income taxes. In theory, households can completely offset the distortionary impact of income taxes by reallocating labor supply to the underground sector. That would happen when the quantity $\left[\frac{1-\sigma}{\sigma}\frac{1}{1-\alpha}\frac{1}{1-\tau_Y}\right]$ completely offset the negative impact of quantities (A)-(F).

Proposition 3 (GDP Elasticity to τ_F) *The elasticity of aggregate GDP $y_t = y_{M,t} + y_{U,t}$ to a* *permanent increase in corporate tax rate τ_F is denoted as ε_{Y,τ_F} and reads:*

$$\varepsilon_{Y,\tau_F} =$$

$$(i) \quad -\frac{\alpha}{1-\alpha}\frac{\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}(1-\tau_Y)}{\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}(1-\tau_Y)(1-\tau_F)\right)} \qquad (<0)$$

$$(ii) \quad -\left[\frac{1}{D_1}\frac{-(1-\tau_Y)(1-\alpha)}{B_M}\bar{\Psi}^{\alpha}\right]+ \qquad (<0)$$

$$(iii) \quad +\left[\frac{1}{D_1}\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\alpha\left(\bar{\Psi}\right)^{\alpha-1}\frac{\partial\bar{\Psi}}{\partial\tau_F}\right]+ \qquad (<0)$$

$$(iv) \quad -\left[(1-\sigma)\lambda_U N_U^{-\sigma}\frac{\partial N_U}{\partial\tau_F}\left(1-(1-\phi)ps\tau_F\right)-\lambda_U N_U^{1-\sigma}(1-\phi)ps\right]+ \qquad \left(\lesseqgtr 0\right)$$

$$(v) \quad +\left[\Omega\frac{\partial\bar{\Psi}}{\partial\tau_F}\frac{1}{D_2}\right]+ \qquad (<0)$$

$$(vi) \quad -\left[(1-\phi)(1-\tau_Y)\lambda_M\frac{1}{D_2}\bar{\Psi}^{\alpha}\right]+ \qquad (<0)$$

$$(vii) \quad -\left[\left[(1-\tau_Y)(1-\tau_F)+\phi\left((1-\tau_F)\tau_Y+\tau_F\right)\right]\lambda_M\alpha\left(\bar{\Psi}\right)^{\alpha-1}\right]\frac{1}{D_2}\frac{\partial\bar{\Psi}}{\partial\tau_F}+ \qquad (>\mathbf{0})$$

$$(viii) \quad +\frac{1-\sigma}{\sigma}\frac{1}{1-\alpha}\frac{1}{1-\tau_F}-\frac{1-\sigma}{\sigma}\frac{1}{(1-ps\tau_F)} \qquad \left(\lesseqgtr \mathbf{0}\right)$$

Proof. See Appendix A. ■

The response of aggregate production to shifts in corporate tax rates τ_F differs, because the direct impact of the tax shock is affected by the the probability that a firm is detected evading, and eventually fined. It reduces, in expected terms, the insurance opportunity that a firm has, compared with households. This is a consequence of assuming that households are not subject to the probability of being detected. We argue that this is not a restrictive assumption. Notice, first, that in equilibrium the households own the firms, and therefore they are, in some sense, subject to the probability of being detected, as owners. On the other hand, tax evasion/elusion is a phenomenon that is much more widespread within the corporate sector.

4.2.2 A feeling for the relative magnitudes, part I: model's prediction

It is also interesting to have a feeling for the relative magnitudes of these elasticities as predicted by the model. **Table 2** presents the figures for the elasticities derived in the previous sections of the paper, under the chosen parametrization. For completeness, the table presents also a sensitivity analysis with respect the parameter ϕ, which represents the degree of substitutability between private and public consumption flows. We mainly focus on this parameter because we consider it being a relevant one for discussing the so called "tax exchange effect"; in addition, figures chosen

Table 2: **Elasticities with and without tax evasion: model's predictions**

	ε_{Y,τ_Y}	$\varepsilon^{\star}_{Y,\tau_Y}$	ε_{Y,τ_F}	$\varepsilon^{\star}_{Y,\tau_F}$
$\phi = 0.706$	**-0,101**	**-0,30**	**-0,0684**	**-0,0807**
$\phi = 0.00$	-0.266	-0.465	-0.0837	-0.0960
$\phi = 1.00$	-0.073	-0.271	-0.0657	-0.0780
$\phi = -1.00$	0.972	0.773	0.0297	0.0175

Notes: ε_{Y,τ_Y} elasticity of aggregate GDP to a 1 percent increase in income tax rate, with tax evasion $\varepsilon^{\star}_{Y,\tau_Y}$: elasticity of aggregate GDP to a 1 percent increase in income tax rate, without tax evasion; ε_{Y,τ_F}: elasticity of aggregate GDP to a 1 percent increase in corporate tax rate, with tax evasion $\varepsilon^{\star}_{Y,\tau_F}$: elasticity of aggregate GDP to a 1 percent increase in corporate tax rate, without tax evasion.

for the other parameters are standard for the literature.

For our parametrization tax evasion almost completely offsets the impact of a 1 percent increase in income and corporate tax rates. The elasticities to a one percent increase in income tax rate equal to $\varepsilon_{Y,\tau_Y} = -0.101$ percent in the tax-evasion-case, and to $\varepsilon^{*}_{Y,\tau_Y} = -0.300$ percent in the no-tax-evasion scenario. Similarly, the elasticities to a one percent increase in corporate tax rate are $\varepsilon_{Y,\tau_F} = -0.0684$ percent (with tax evasion) and $\varepsilon^{*}_{Y,\tau_F} = -0.0807$ percent (without tax evasion). The elasticities under tax evasion (ε_{Y,τ_Y} and ε_{Y,τ_F}) are, in absolute value smaller than those computed for the corresponding economy without tax evasion ($\varepsilon^{*}_{Y,\tau_Y}$ and $\varepsilon^{*}_{Y,\tau_F}$). These results are in line with our interpretation stressing the smoothing effect of the underground sector.

Now, when private goods complement government consumption, i.e. $\phi > 0$, Table 2 shows that the tax policy effect is reduced. In the case of wasteful public expenditure ($\phi = 0.00$), the impact of a 1 percent increase in income or corporate tax rates has a relatively stronger impact (i.e. more contractionary), whereas the aggregate output contraction is much smaller when the private and public consumption are complement goods ($\phi = 1.00$). On the contrary, when the private goods substitute government consumption ($\phi = -1.00$), a tax increase is not anymore "Keynesian". In this case the agents of this economy manage to avoid the contractionary effect of the policy, leading the economy to shift from the market to the underground activities and pushing, from this way, consumption and investment. The different response is a consequence of what is often time described as "the fiscal exchange" i.e. the share of collected tax revenues that tax-payer consumer collects back as a merit good.

These results, even in this simplified structure of the model where all the agents are equally taxpayers and evaders, should not be much surprising if we consider the size of the hidden economy and the relative tax evasion. The fact that the public expenditure is an important variable to determine tax evasion is not only because of the burden of taxation necessary for budget balancing, but also for the consumer's choice to evade throughout the perceived fiscal-exchange. For instance, a recent survey of Isae 2005 (Institute of Studies for Economic Analysis) has highlighted that a share of 66% of Italian taxpayers in the "fiscal exchange" with government, believe that they are in operating loss; giving to the government more in taxed income than that obtain from it in the form of public goods and services.

4.2.3 A feeling for the relative magnitudes, part II: empirical estimates

Now, once we have figures for the different elasticities as predicted by our theoretical model, it would interesting to compare these with the corresponding empirical estimates. Because we are not aware, to the best of our knowledge, to contribution explicitly analyzing this aspect, we attempt below a measurement exercise. It is important to stress that the purpose of this exercise it to have a feeling for the relative magnitudes; it is not our intention to perform a fully-fledged econometric exercise.

Now, given this premise, relying on the ISTAT database (i.e. the Italian Statistical Institute; www.istat.it/enslish) we can construct two measures of the GDP: one includes the contribution of the underground sector, while the second does not. The second measure is constructed using, as a proxy for the underground sector, the tax evasion associated to the Value Added Tax Taxable basis, provided by the Agenzia delle Entrate (http://www.agenziaentrate.it).

The elasticities are obtained performing cointegrating long-run regressions adding a trend component to the equations. The coefficients are statistically significant and the variable residuals are statistically satisfactory. ADF tests on equation residuals indicate stationary relations at the 0.05 levels. The residual serial correlation LM test shows that the null of no serial correlation up to lag 3 is not rejected.

Table 3, next, presents empirical estimates of the aggregate GDP elasticities to a one percentage increase in income (top panel) or corporate (bottom panel) tax rates, for two scenarios: one with tax evasion (columns on the left), and one without (columns on the right). The table also compares the empirical estimates with the model's prediction. Appendix B accurately describes the data sources and the construction of variables.

Table 3: **GDP Elasticities: empirical estimates and model predictions**

GDP ELASTICITY W.R.T ONE PERCENT INCREASE IN INCOME TAX RATE

with tax evasion		*without* tax evasion	
Model's prediction	Empirical estimates	Model's prediction	Empirical estimates
-0.10	-0.15 (0.07)	-0.30	-0.33 (0.10)

GDP ELASTICITY W.R.T ONE PERCENT INCREASE IN CORPORATE TAX RATE

with tax evasion		*without* tax evasion	
Model's prediction	Empirical estimates	Model's prediction	Empirical estimates
-0.07	-0.04 (0.03)	-0.08	-0.17 (0.04)

Notes: Top panel includes the GDP elasticity following a one percent increase in income tax rate; next, the bottom panel includes the GDP elasticity following a one percent increase in corporate tax rate. For both panels, the right column compares the corresponding elasticities without tax evasion, while the left one column compares the elasticities with tax evasion. Numbers in parenthesis represents the standard error; all estimates are significantly different from zero.

The table suggests (third column) that the estimated elasticity of aggregate GDP with respect income tax rates equals to -0.33; these figures are quantitatively and qualitatively consistent with the empirically estimated figures, presented on the right end side.

The table also suggests (sixth column) that the that the estimated elasticity of aggregate GDP with respect a measure of corporate tax rates ranges between -0.15 and -0.17 percent; again, we can consider that these figures are qualitatively consistent with the empirical estimates.

In general, we think it is important to notice that this parsimonious macroeconomic model is capable of predicting figures for the GDP elasticities to a one percent increase in income and corporate tax rates that are, at least qualitatively, consistent with the corresponding empirical figures.

In addition, the relatively smaller, in absolute value, elasticities under tax evasion are perfectly consistent with consumption and income smoothing. This seems supporting the claim that tax evasion and the informal economy offer a channel for insuring income and consumption patters from distortions generated by fiscal policy. In this sense tax evasion can be interpreted as an income smoothing device available to households and firms. It is like saying that the government chooses in fact the statutory tax rates, while the effective tax rates are endogenously chosen by households and firms by relying on the additional dimension represented by tax evasion.

4.3 Laffer curve under tax evasion

One of the most controversial issues in tax policy analysis is whether a tax cut will boost economic activity to such an extent that the government's budget actually improves (this often referred to as a Laffer's Curve Effect).[28]

Our contribution examines the possibility that tax evasion and underground economy give a rational justification for a variant of a Laffer curve. In this context we deal with a Laffer surface, given that we consider two tax rates. The Laffer surface under tax evasion is defined as the function $\mathfrak{L}^*(\tau_Y, \tau_F) : [0,1] \times [0,1] \mapsto \mathbb{R}_+$ such that

$$\mathfrak{L}^*(\tau_Y, \tau_F) = ((1-\tau_Y)(1-\tau_F)\tau_Y + \tau_F)(K^*(\tau_Y, \tau_F))^{\alpha}(N_M^*(\tau_Y, \tau_F))^{1-\alpha} + \tau_F ps (N_U^*(\tau_Y, \tau_F))^{1-\sigma},$$

where $\tau_Y, \tau_F \in [0,1]$ and $K^*(\tau_Y, \tau_F)$, $N_M^*(\tau_Y, \tau_F)$, $N_U^*(\tau_Y, \tau_F)$ denote the equilibrium quantities, evaluated at the stationary state (derived in Proposition 1). The first quantity $((1-\tau_Y)(1-\tau_F)\tau_Y + \tau_F)(K^*(\tau_Y, \tau_F))^{\alpha}(N_M^*(\tau_Y, \tau_F))^{1-\alpha}$ represents revenues collected from the regular demand side and the supply sides of the economy, while the second part $\tau_F ps (N_U^*(\tau_Y, \tau_F))^{1-\sigma}$ represents revenues collected, in expected terms, from the underground sector. We parameterize the model for the Italian economy, given the availability of a good data-set for the underground economy; then **Figure 2** plots the bi-dimensional Laffer curve (surface) for our economy.

The Laffer curve under tax evasion (upper panel) is *almost always* below the corresponding curve computed for a 100 percent regular economy. To better appreciate this result, it is convenient to distinguish between (i) movement along the Laffer's curve, and (ii) shifts of the curve itself. The underground sector, and therefore tax evasion allows for shifts of the curve itself. For example, after a tax cut the resource reallocation mechanism moves resources above the ground, and therefore increases the tax base, shifting the Laffer curve upward. In addition, the model with tax evasion also predicts the movement along the curve, which represents the increase of the regular tax base. In addition, notice that for *sufficiently high* tax rates government revenues go to zero in an economy

[28] A Laffer curve can be defined as a curve which supposes that for a given economy there is an optimal income tax level to maximize tax revenues. If the income tax level is set below this level, raising taxes will increase tax revenue. And if the income tax level is set above this level, then lowering taxes will increase tax revenue. Although the theory claims that there is a single maximum and that the further you move in either direction from this point the lower the revenues will be, in reality this is only an approximation.

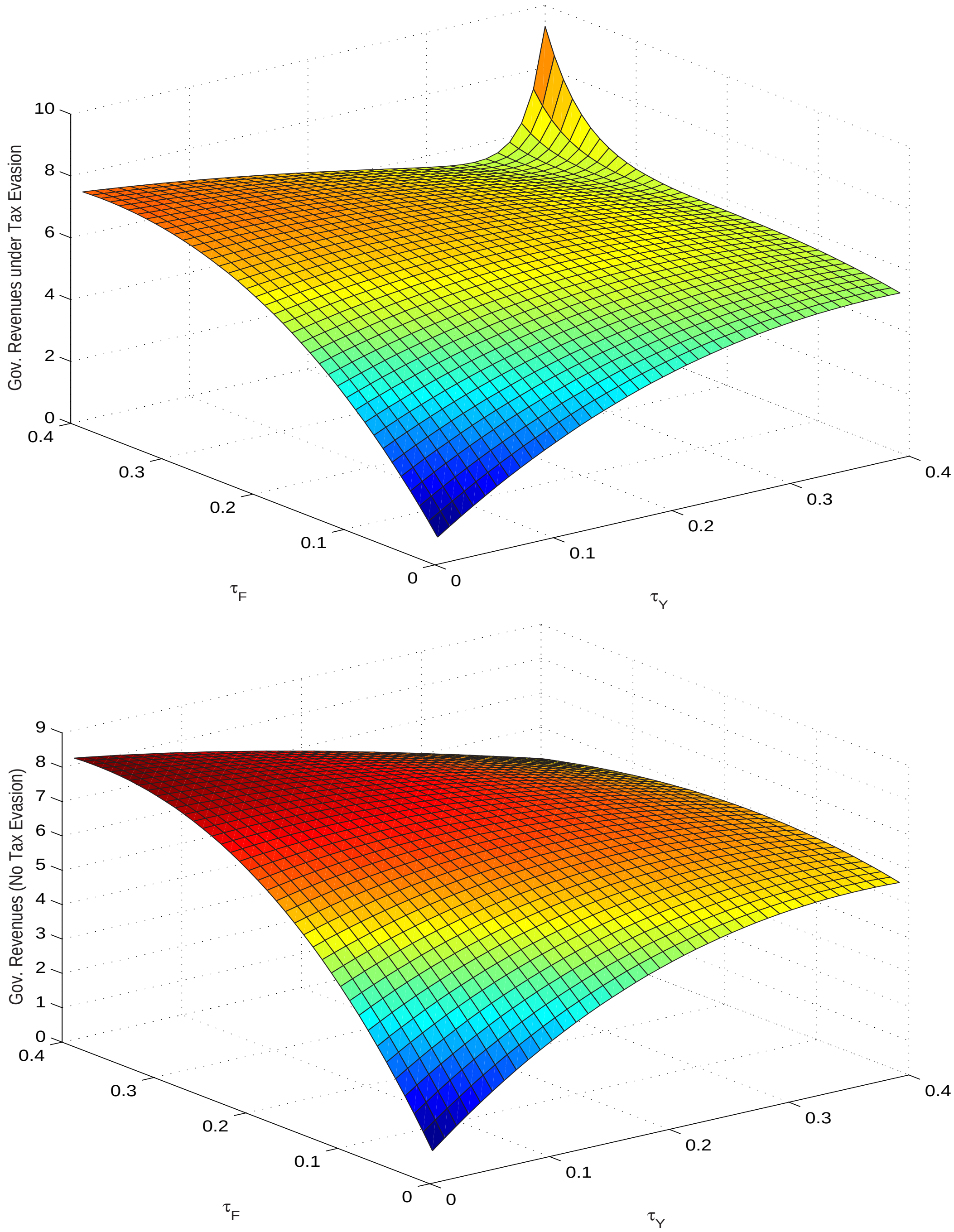

Figure 2: **Laffer Curve with (without) Tax Evasion**. Left Panel: Laffer curve under Tax Evasion; Right Panel: Laffer curve without Tax Evasion. The quantities τ_Y and τ_F denote income and corporate tax rates. The model's parametrization is included in Section 4.1

without tax evasion (that is the Laffer curve in the top panel of Figure 2 is flat at zero for (τ_Y, τ_F) large); with tax evasion, on the contrary, government revenues begin rising again for very large tax rates (that is the upward sloping tail of the Laffer curve in the bottom panel of Figure 2).

The "*upward sloping*" tail of the Laffer curve under tax evasion (bottom panel of Figure 2) is a consequence of the expansionary effects following "too severe" contractionary fiscal policies (more details to come). Recall that an increase in the tax rate induces the economy to reallocate resources to the underground sector. The higher the tax rate, the stronger would be, *ceteris paribus*, the reallocation toward the informal sector. This reallocation increases households' disposable income, because underground produced revenues are not taxed. Now, if this resource reallocation is sufficiently large, an increase in the tax rate produces an increase in capital accumulation as well (recall that consumption and investment are normal goods, and therefore an increase in income is allocated between both goods). In this case the capital accumulation would be financed though underground-produced revenues; in this respect it can be argued that evaded taxes represent a sort of (illegal) internal finance for the firms.

4.3.1 Threshold for expansionary effects of contractionary fiscal policies

In this context it is interesting to compute the critical level of household and firm tax rate that triggers the reallocation to underground labor, and, by this end, the appearance of expansionary effects of contractionary fiscal policies. The threshold in this context is defined as the set

$$\mathcal{T} = \left\{ \left(\tau_Y^0, \tau_F^0\right) : RV\left(\tau_Y^0 + \varepsilon, \tau_F^0 + \varepsilon\right) \geq RV\left(\tau_Y^0, \tau_F^0\right), \varepsilon > 0 \right\},$$

where RV denotes government revenues at the steady state. $\mathcal{T}$ defines the set of tax rates beyond which a further increase in tax rate triggers a reallocation toward the underground sector capable to increase the overall government revenues. In other words if a pair (τ_Y, τ_F) falls in the $\mathcal{T}$ set, it triggers the aforementioned mechanism, which, in turn, pulls the economy toward a tax evasion-driven expansion. It is important to underline, however, that we would observe an increase of regular GDP and of government revenues collected from the regular economy; here the role of the underground sector is being the spark that ignites the mechanism.

Figure 3 below presents this threshold set for the baseline parametrization (the solid line with circles) and for different parameter combinations. The threshold is operationally derived by computing the pairs $\left(\tau_Y^0, \tau_F^0\right)$ at which the numerical gradient of Laffer curve $\mathfrak{L}^*\left(\tau_Y, \tau_F\right)$ is zero, and the numerical Hessian is positive definite.

The figure suggests that expansionary effects of contractionary fiscal policies arise at lower levels of income and corporate tax rates when the regular labor becomes increasingly costly ($\uparrow B_M$), when underground production becomes more and more flexible ($\downarrow \sigma$). On the contrary, these effects are more unlikely to happen when underground labor is more risky ($\uparrow B_U$).

5 Conclusions

This paper studies equilibrium effects of fiscal policy disturbances within a dynamic general equilibrium model where tax evasion and underground activities are explicitly incorporated. It is here shown that an underground sector mitigates the distortionary impact of fiscal policies, while lessening the drop (and the rise) of aggregate production after restrictive (expansionary) tax changes. In this sense the theoretical discussion suggests that tax evasion and underground economy can be seen as an economic mechanism that rationalizes expansionary effects to contractionary fiscal policies.

In summary, the paper stresses that fiscal policies may be significantly affected by tax evasion. A simple econometric exercise presented in the paper shows that the GDP elasticities to an increase in tax rates under tax evasion are very close to zero, while those computed without tax evasion are negative, consistently with the predictions of the neoclassical growth model. The almost zero elasticity value under tax evasion is here consistent with consumption and income smoothing. Tax evasion and the informal economy offer, in other words, a channel for insuring income and consumption patters from fluctuations generated by fiscal policy. In this sense tax evasion can be interpreted as a smoothing device available to households and firms. It is like saying that the government chooses in fact statutory tax rates, while the effective tax rates are endogenously chosen by households and firm by relying on the additional dimension represented by tax evasion.

We think that these are crucial observations that have several implications to be exploited from a theoretical perspective and a policy perspective.

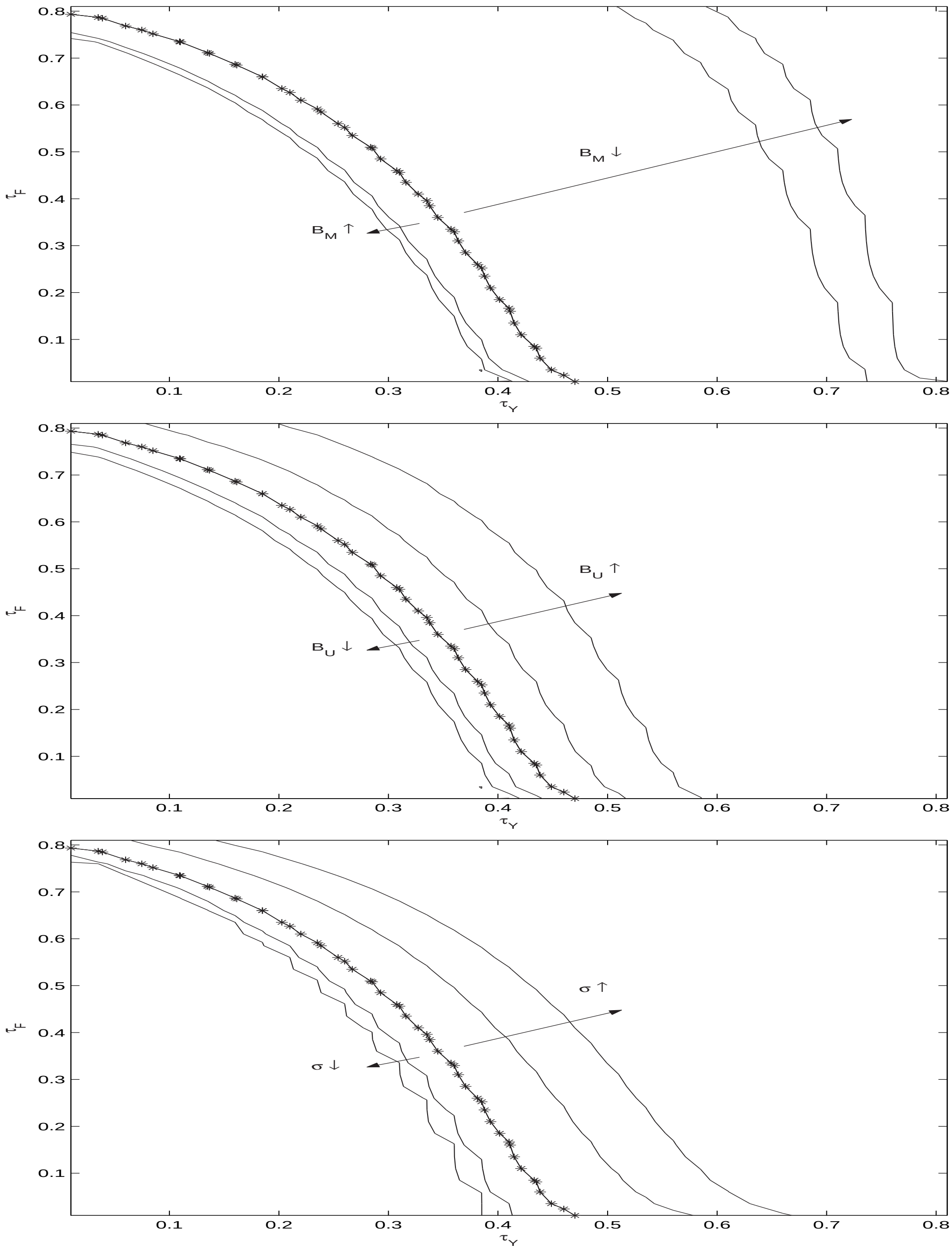

Figure 3: **Threshold for expansionary effects of recessionary fiscal policies**. The panels present the threshold levels of tax rates (τ_Y, and τ_F) beyond which the model displays expansionary effects of recessionary fiscal policies. Solid line with circles: baseline parameterization; solid lines: sensitivity analysis; the dashed arrow denotes the direction in which the threshold set $\mathcal{T}$ moves by perturbing the parameters' space. Upper-left panel: sensitivity with respect to regular labor disutility parameter B_M; upper-right panel: sensitivity with respect to the degree of substitutability between government and private consumptions ϕ; bottom-left panel: sensitivity with respect to the idiosyncratic cost of working in the underground sector B_U; bottom-right panel: sensitivity with respect elasticity of underground labor σ.

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Appendix A: Outline of the proofs

Proof of Proposition 1 (deterministic stationary state): deterministic stationary state is obtained imposing certainty equivalence, assuming for alla variables that $x_t = x_{t+1}$, for alla t. Deterinistic stationary state is derived under market clearing conditions for all markets. Now, given the relative prices for production inputs (marginally priced from firm profits maximization) that are reported below for convenience,

$$w_M = (1-\tau_F)(1-\alpha)\lambda_M K^{\alpha} N_M^{-\alpha}$$
$$w_U = (1-ps\tau_F)(1-\sigma)\lambda_U N_U^{-\sigma}$$
$$r = (1-\tau_F)\alpha\lambda_M K^{\alpha-1} N_M^{1-\alpha},$$

the model´s equilibrium is characterized by the following conditions:

$$\begin{aligned}
&B_M = (C+\varphi C_G)^{-1}(1-\tau_Y)(1-\tau_F)(1-\alpha)\lambda_M K^{\alpha} N_M^{-\alpha}\\
&B_U + B_M = (C+\varphi C_G)^{-1}(1-ps\tau_F)(1-\sigma)\lambda_U N_U^{-\sigma}\\
&C + \Omega K = (1-\tau_Y)(1-\tau_F)\lambda_M K^{\alpha} N_M^{1-\alpha} + (1-ps\tau_F)(1-\sigma)\lambda_U N_U^{1-\sigma} + \pi\\
&\pi = (1-ps\tau_F)\lambda_U N_U^{1-\sigma}\sigma\\
&C_G = \lambda_M K^{\alpha} N_M^{1-\alpha}\left[(1-\tau_F)\tau_Y + \tau_F\right] + \tau_F ps\lambda_U N_U^{1-\sigma}\\
&1 = \beta\left((1-\tau_Y)r + 1 - \Omega\right).
\end{aligned}$$

Step 1: stationary equilibrium for $\frac{K}{N_M}$. This quantity is quickly derived from the Euler equation combined with the marginal remuneration for physical capital (reported below for reader´s convenience):

$$\begin{aligned}
(C+\phi C_G)^{-1} &= \beta\mathbb{E}_t(C+\phi C_G)^{-1}\left((1-\tau_Y)r + 1 - \Omega\right)\\
r &= (1-\tau_F)\alpha\lambda_M K^{\alpha-1} N_M^{1-\alpha}\\
&\downarrow\\
\frac{K}{N_M} &= \left(\frac{\alpha\lambda_M(1-\tau_Y)(1-\tau_F)}{\beta^{-1}-1+\Omega}\right)^{\frac{1}{1-\alpha}} = \bar{\Psi}
\end{aligned}$$

Step 2: stationary equilibrium for N_U. Combining the first order conditions with respect to N_M and N_U (reported below for reader´s convenience), we obtain:

$$B_M = (C+\varphi C_G)^{-1}(1-\tau_Y)(1-\tau_F)(1-\alpha)\lambda_M K^{\alpha} N_M^{-\alpha}$$

$$B_U + B_M = (C+\varphi C_G)^{-1}(1-ps\tau_F)(1-\sigma)\lambda_U N_U^{-\sigma}$$

$$\downarrow$$

$$\frac{B_M}{(B_U+B_M)}\frac{(1-ps\tau_F)}{(1-\tau_Y)(1-\tau_F)}\frac{\lambda_U}{\lambda_M} = \frac{(1-\alpha)\left(k_t^i\right)^{\alpha}\left(n_{M,t}^i\right)^{-\alpha}}{(1-\sigma)\, n_U^{-\sigma}}$$

After several steps of algebra, and isolating terms including tax rates, we have the final expression for stationary value for underground labor.

$$N_U = \left[\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\right)^{-\frac{\alpha}{\sigma(1-\alpha)}}\left(\frac{B_M}{B_U+B_M}\frac{\lambda_U}{\lambda_M}\right)^{\frac{1}{\sigma}}\left(\frac{1-\sigma}{1-\alpha}\right)^{\frac{1}{\sigma}}\right]\times$$
$$\times\left[\left(\frac{1}{1-\tau_Y}\right)^{\frac{1}{\sigma(1-\alpha)}}\left(\frac{1}{1-\tau_F}\right)^{\frac{1}{\sigma(1-\alpha)}}(1-ps\tau_F)^{\frac{1}{\sigma}}\right]$$

Step 3: stationary equilibrium for N_M. First, combine the aggregate feasibility constraint of the economy with equilibrium profits equilibrium government consumption, and marginal prices for production inputs; it yields

$$C+\phi C_G = (1-\tau_Y)(1-\tau_F)\lambda_M K^{\alpha} N_M^{1-\alpha} + (1-ps\tau_F)\lambda_U N_U^{1-\sigma} +$$
$$-\Omega K + \phi\lambda_M K^{\alpha} N_M^{1-\alpha}\left[(1-\tau_F)\tau_Y+\tau_F\right] + \phi\tau_F ps\lambda_U N_U^{1-\sigma}$$

Then, factorizing out N_M yields

$$C+\phi C_G = N_M\left[(1-\tau_Y)(1-\tau_F)\lambda_M\left(\frac{K}{N_M}\right)^{\alpha} - \Omega\frac{K}{N_M} + \phi\lambda_M\left(\frac{K}{N_M}\right)^{\alpha}\left[(1-\tau_F)\tau_Y+\tau_F\right]\right] +$$
$$+\lambda_U N_U^{1-\sigma}\left[1-(1-\phi)\,ps\tau_F\right]$$

Now, the strategy is to substitute the previous expression into the first order condition for N_M:

$$(C+\varphi C_G) = \frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\left(\frac{K}{N_M}\right)^{\alpha} = \frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\left(\bar{\Psi}\right)^{\alpha}$$

Noticing that the RHS is constant in the stationary state implies

$$N_M \left[(1-\tau_Y)(1-\tau_F)\lambda_M \left(\frac{K}{N_M}\right)^{\alpha} - \Omega \frac{K}{N_M} + \phi\lambda_M \left(\frac{K}{N_M}\right)^{\alpha} [(1-\tau_F)\tau_Y + \tau_F] \right] = \frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M} \left(\bar{\Psi}\right)^{\alpha} - \lambda_U N_U^{1-\sigma} [1-(1-\phi)ps\tau_F].$$

Notice that also in this expression the RHS is constant because N_U has been derived before; this implies

$$N_M = \frac{\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M} \left(\bar{\Psi}\right)^{\alpha} - \lambda_U N_U^{1-\sigma} [1-(1-\phi)ps\tau_F]}{(1-\tau_Y)(1-\tau_F)\lambda_M \left(\frac{K}{N_M}\right)^{\alpha} - \Omega \frac{K}{N_M} + \phi\lambda_M \left(\frac{K}{N_M}\right)^{\alpha} [(1-\tau_F)\tau_Y + \tau_F]},$$

or in a more compact notation

$$N_M = \frac{\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M} \left(\bar{\Psi}\right)^{\alpha} - \lambda_U N_U^{1-\sigma} [1-(1-\phi)ps\tau_F]}{[(1-\tau_Y)(1-\tau_F) + \phi((1-\tau_F)\tau_Y + \tau_F)]\lambda_M \left(\bar{\Psi}\right)^{\alpha} - \Omega\bar{\Psi}}.$$

Step 4. Stationary value for K; quickly derived from the Euler equation

$$K = \bar{\Psi} N_M,$$

where N_M has been derived above.

In summary: the revised steady state is the following:

$$\begin{aligned}
\overline{N}_M &= \frac{\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M} \left(\bar{\Psi}\right)^{\alpha} - \lambda_U N_U^{1-\sigma} [1-(1-\phi)ps\tau_F]}{[(1-\tau_Y)(1-\tau_F) + \phi((1-\tau_F)\tau_Y + \tau_F)]\lambda_M \left(\bar{\Psi}\right)^{\alpha} - \Omega\bar{\Psi}} \\
& \overline{N}_U = \left[\left(\frac{\alpha\lambda_M}{\beta^{-1} - 1 + \Omega}\right)^{-\frac{\alpha}{\sigma(1-\alpha)}} \left(\frac{B_M}{B_U + B_M}\frac{\lambda_U}{\lambda_M}\right)^{\frac{1}{\sigma}} \left(\frac{1-\sigma}{1-\alpha}\right)^{\frac{1}{\sigma}} \right] \times \\
& \left[\left(\frac{1}{1-\tau_Y}\right)^{\frac{1}{\sigma(1-\alpha)}} \left(\frac{1}{1-\tau_F}\right)^{\frac{1}{\sigma(1-\alpha)}} (1-ps\tau_F)^{\frac{1}{\sigma}} \right] \\
\overline{K} &= \left(\frac{\alpha(1-\tau_Y)(1-\tau_F)}{\beta^{-1} - 1 + \Omega}\right)^{\frac{1}{1-\alpha}} \overline{N}_M
\end{aligned}$$

This completes the sketch of the proof for Proposition 1 □

Outline of Proof of Proposition 2 (GDP elasticity with respect income tax rate)

Strategy of the proof is to derive an equilibrium expression for the logarithm of aggregate GDP as a function of tax rates, and then differentiate it with respect each tax rate. It is in this context convenient to first compute the logarithm of underground produced GDP and of the regular component.

Consider, first, the equilibrium expression for underground component of aggregate GDP; taking logs, factorizing out tax rates, and, ignoring, for the sake of simplicity, quantities that does not include tax rates, it yields:

$$\ln \overline{Y}_U(\tau_Y, \tau_F) = \ln(1-\tau_Y)\frac{1-\sigma}{\sigma}\left[-1-\frac{\alpha}{(1-\alpha)}\right] + \ln(1-\tau_F)\left[-\frac{1-\sigma}{\sigma}-\frac{(1-\sigma)\alpha}{\sigma(1-\alpha)}\right] + \frac{1-\sigma}{\sigma}\ln(1-ps\tau_F).$$

Consider, then, the equilibrium expression for regular production, take logs, and we obtain $\log y_M = \alpha \log k + (1-\alpha)\log n_M$. It is convenient to separately study the elasticity of regular labor service and of capital stock to tax rate. Begin with the natural logarithm of regular labor services:

$$\begin{aligned}\ln \overline{N}_M &= \ln\left\{\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\left(\bar{\Psi}\right)^{\alpha} - \lambda_U N_U^{1-\sigma}\left[1-(1-\phi)ps\tau_F\right]\right\} + \\ &\quad - \ln\left\{\left[(1-\tau_Y)(1-\tau_F) + \phi\left((1-\tau_F)\tau_Y + \tau_F\right)\right]\lambda_M\left(\bar{\Psi}\right)^{\alpha} - \Omega\bar{\Psi}\right\}.\end{aligned}$$

Differentiating it with respect to income tax rate τ_Y, yields:

$$\begin{aligned}\frac{\partial \ln N_M(\tau_Y, \tau_F)}{\partial \tau_Y} &= \frac{\frac{-(1-\tau_F)(1-\alpha)}{B_M}\bar{\Psi}^{\alpha} + \frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\frac{\partial \bar{\Psi}^{\alpha}}{\partial \tau_Y} - (1-\sigma)\lambda_U N_U^{-\sigma}\frac{\partial N_U}{\partial \tau_Y}\left[1-(1-\phi)ps\tau_F\right]}{\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\left(\bar{\Psi}\right)^{\alpha} - \lambda_U N_U^{1-\sigma}\left[1-(1-\phi)ps\tau_F\right]} + \\ &\quad - \frac{\left[(1-\tau_Y)(1-\tau_F) + \phi\left((1-\tau_F)\tau_Y + \tau_F\right)\right]\lambda_M \alpha\left(\bar{\Psi}\right)^{\alpha-1}\frac{\partial \bar{\Psi}}{\partial \tau_Y} + \left[-(1-\tau_F) + \phi(1-\tau_F)\right]\lambda_M\left(\bar{\Psi}\right)^{\alpha} - \Omega\frac{\partial \bar{\Psi}}{\partial \tau_Y}}{\left[(1-\tau_Y)(1-\tau_F) + \phi\left((1-\tau_F)\tau_Y + \tau_F\right)\right]\lambda_M\left(\bar{\Psi}\right)^{\alpha} - \Omega\bar{\Psi}}\end{aligned}$$

After several steps of algebra, it can be shown that the elasticity of regular labor services to an increase in income tax rate can be written as follows:

$$
\begin{aligned}
\varepsilon_{N_M,\tau_Y} = \quad & -\left[\tfrac{1}{D_1}\tfrac{(1-\tau_F)(1-\alpha)}{B_M}\bar{\Psi}^{\alpha}\right]+ && (<0)\\
& +\left[\tfrac{1}{D_1}\tfrac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\alpha\left(\bar{\Psi}\right)^{\alpha-1}\tfrac{\partial\bar{\Psi}}{\partial\tau_Y}\right]+ && (<0)\\
& +\left[\tfrac{1}{D_1}\left(1-\sigma\right)\lambda_U N_U^{-\sigma}\tfrac{\partial N_U}{\partial\tau_Y}\left[1-\left(1-\phi\right)ps\tau_F\right]\right]+ && (>\mathbf{0})\\
& +\left[\Omega\tfrac{\partial\bar{\Psi}}{\partial\tau_Y}\tfrac{1}{D_2}\right]+ && (<0)\\
& -\left[\left(1-\phi\right)\left(1-\tau_F\right)\lambda_M\tfrac{1}{D_2}\bar{\Psi}^{\alpha}\right]+ && (>\mathbf{0})\\
& -\left[\left(\left(1-\tau_Y\right)\left(1-\tau_F\right)+\phi\left(\left(1-\tau_F\right)\tau_Y+\tau_F\right)\right)\lambda_M\alpha\left(\bar{\Psi}\right)^{\alpha-1}\right]\tfrac{1}{D_2}\tfrac{\partial\bar{\Psi}}{\partial\tau_Y} && (>\mathbf{0})
\end{aligned}
$$

Next step is to compute $\log K$

$$
\begin{aligned}
K &= \left[\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_Y\right)\left(1-\tau_F\right)\right]^{\frac{1}{1-\alpha}}\overline{N}_M\\
\ln K &= \frac{1}{1-\alpha}\ln\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_Y\right)\left(1-\tau_F\right)\right)+\ln\overline{N}_M
\end{aligned}
$$

and the corresponding elasticity with respect to τ_Y

$$
\begin{aligned}
\frac{\partial\ln K}{\partial\tau_Y} &= \frac{\partial}{\partial\tau_Y}\frac{1}{1-\alpha}\ln\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_Y\right)\left(1-\tau_F\right)\right)+\frac{\partial\ln\overline{N}_M}{\partial\tau_Y}\\
\varepsilon_{K,\tau_Y} &= -\frac{1}{1-\alpha}\frac{\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_F\right)}{\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_Y\right)\left(1-\tau_F\right)\right)}+\varepsilon_{N_M,\tau_Y}
\end{aligned}
$$

And finally we compute:

$$
\begin{aligned}
\frac{\partial\ln Y_M}{\partial\tau_Y} &= \alpha\frac{\partial\ln K}{\partial\tau_Y}+\left(1-\alpha\right)\frac{\partial\ln N_M}{\partial\tau_Y}\\
\varepsilon_{Y_M,\tau_Y} &= \alpha\varepsilon_{K,\tau_Y}+\left(1-\alpha\right)\varepsilon_{N_M,\tau_Y}\\
&= -\frac{\alpha}{1-\alpha}\frac{\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_F\right)}{\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_Y\right)\left(1-\tau_F\right)\right)}+\alpha\varepsilon_{N_M,\tau_Y}+\left(1-\alpha\right)\varepsilon_{N_M,\tau_Y}\\
&= -\frac{\alpha}{1-\alpha}\frac{\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_F\right)}{\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_Y\right)\left(1-\tau_F\right)\right)}+\varepsilon_{N_M,\tau_Y}
\end{aligned}
$$

Approximating the total output as $logy_{TOT}\simeq logy_M+logy_U$, corresponding elasticity with respect income tax rate reads:

$$\varepsilon_{Y,\tau_Y} = -\frac{\alpha}{1-\alpha}\frac{\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_F\right)}{\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}\left(1-\tau_Y\right)\left(1-\tau_F\right)\right)} + \varepsilon_{N_M,\tau_Y} + \varepsilon_{Y_U,\tau_Y}$$

To conclude, combining now all expressions into the previous one and we have

$$\begin{aligned}
\varepsilon_{Y,\tau_Y} = & \\
& -\frac{\alpha}{1-\alpha}\frac{\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}(1-\tau_F)}{\left(\frac{\alpha\lambda_M}{\beta^{-1}-1+\Omega}(1-\tau_Y)(1-\tau_F)\right)} && (<0)\\
& -\left[\frac{1}{D_1}\frac{(1-\tau_F)(1-\alpha)}{B_M}\bar{\Psi}^{\alpha}\right] + && (<0)\\
& +\left[\frac{1}{D_1}\frac{(1-\tau_Y)(1-\tau_F)(1-\alpha)}{B_M}\alpha\left(\bar{\Psi}\right)^{\alpha-1}\frac{\partial\bar{\Psi}}{\partial\tau_Y}\right] + && (<0)\\
& -\left[\frac{1}{D_1}\left(1-\sigma\right)\lambda_U N_U^{-\sigma}\frac{\partial N_U}{\partial\tau_Y}\left[1-\left(1-\phi\right)ps\tau_F\right]\right] + && (<0)\\
& +\left[\Omega\frac{\partial\bar{\Psi}}{\partial\tau_Y}\frac{1}{D_2}\right] + && (<0)\\
& -\left[\left(1-\phi\right)\left(1-\tau_F\right)\lambda_M\frac{1}{D_2}\bar{\Psi}^{\alpha}\right] + && (>\mathbf{0})\\
& -\left[\left(\left(1-\tau_Y\right)\left(1-\tau_F\right)+\phi\left(\left(1-\tau_F\right)\tau_Y+\tau_F\right)\right)\lambda_M\alpha\left(\bar{\Psi}\right)^{\alpha-1}\right]\frac{1}{D_2}\frac{\partial\bar{\Psi}}{\partial\tau_Y} + && (>\mathbf{0})\\
& +\left[\frac{1-\sigma}{\sigma}\frac{1}{1-\alpha}\frac{1}{1-\tau_Y}\right] && (>\mathbf{0})
\end{aligned}$$

This completes the sketch of the proof for Proposition 2 □

Outline of Proof of Proposition 3 (GDP elasticity with respect corporate tax rate)

Strategy is the same one followed for computing the GDP elasticity with respect to income tax rate (i.e. Proposition 2). The only difference consists in differentiating the key variables with respect to corporate tax rate rather than with respect to income tax rate. This completes the sketch of the proof for Proposition 3 □

Appendix B: estimation output and data sources

The data set refers to annual time series from Italian National Account over the period 1980 - 2005.

The estimated equations include the following variables: the real aggregate gross domestic output GDP , the total Revenues on current and capital bases Tax (i.e. the sum of Individual Direct Income Taxes, Indirect taxes and Social security contributions), a measure of Corporate tax burdens TaxF (i.e. Taxes on firms "IRPEG and IRES", local and regional Taxes "ILOR, Irap, other taxes") the Total Added value VTOT.

The source of these time series is the Italian Statistical Institute (ISTAT 2006, Consolidates Account of Public sector, various tables). All the variables are deflated by the implicit GDP deflator.

To estimate tax rate elasticities the following variables have been constructed: The "income tax rate " with tax evasion Tau = Tax / GDP and without Taur = Tax / GDPREG The "corporate tax rate" with evasion Tc5 = TaxF / VTOT and without Tc6 = TaxF / VREG

Total GDP includes the measure of "underground economy", but official estimates from ISTAT to obtain the exhaustiveness of estimates are available only starting from 2001. We define GDPREG and VREG the "regular" components (i.e. total GDP and added value minus tax evasion). About the irregular component , the source of our measure of underground economy is Marigliani - Pisani 2006 . The evasion of "VAT taxable basis" represents a proxy for tax evasion.

Appendix C: Underground economy Vs home production

It is interesting to compare the theoretical structure of our model with household (home) production models. We focus on three selected issues: the commodities' number and their substitutability, the financing of capital investment and the insurance opportunities offered by home production and underground activities. First, consider the number of consumption goods and their substitutability. In the home production class of models there exist two goods, denoted as market and non-market commodities, each of which is produced with a sector specific technology. In addition, the preference specification allows for different degrees of substitutability between market and non-market goods. In contrast, in the model with an underground sector there exists only one homogenous good, which, however, is produced using two different technologies: one associated with the regular sector, and the other with the underground sector. In this environment it is natural to focus on the case of perfect substitutability between regularly-produced final output and underground-produced output. The second difference concerns the financing of investments. In home production models, only regularly-produced goods can be consumed and invested, either into regular capital or into non-market capital. There is no use for home production output other than consumption - it cannot be sold or transformed into capital, for example, the way that regularly-produced output can. In the underground economy model, however, there exists only one capital stock (invested in the regular sector), but regularly-produced and underground-produced output can be transformed into regular capital without any adjustment cost. The underground sector offers an additional channel for financing capital stock accumulation, and an additional dimension along which firms can employ the available labor supply. Summarizing, while the home production model is a legitimate two sector model, our model could be more appropriately defined as a two technology model, since the same good is produced using two different technologies. In addition, an underground sector offers profit smoothing opportunities for firms and insurance opportunities for consumers. More precisely, firms can smooth their profits by a proper allocation of labor demand between the two sectors, on a period by period basis. In addition, consumers can smooth not only consumption, by substituting over time consumption and investments, but also income, by allocating their labor supply across sectors, on a period by period basis. This mechanism is absent in models with home production.