DYNAMIC TAX EVASION WITH AUDITS BASED ON CONSPICUOUS CONSUMPTION

Rosella Levaggi, Università di Brescia
Francesco Menoncin, Università di Brescia

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Keywords: dynamic tax evasion, targeted audits, conspicuous consumption
Dynamic Tax Evasion with Audits based on Conspicuous Consumption

Rosella Levaggi∗ and Francesco Menoncin†

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Abstract

We solve the problem of a representative agent who maximises the expected present utility of his intertemporal consumption under the assumption that an optimal fraction of his wealth is hidden to the tax authorities (we show conditions under which evasion is expedient). Evasion affects the capital dynamics in two ways: the growth rate of capital increases because some taxes are not paid, but when caught evading the consumer has to pay a fee (proportional to evasion). Consumption can be allocated between ordinary goods and so-called conspicuous goods. The latter are used by the Government for targeting the audit, since they are considered like an indicator of consumer’s wealth. In fact, the probability to be caught is a function of the distance between the actual and the presumed consumption in conspicuous goods. We find a closed form solution to the dynamic optimization problem and show how fiscal and audit parameters affect the optimal evasion and the optimal allocation between the two consumptions.

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∗Corresponding author. University of Brescia, Department of Economics, Via S. Faustino, 7/b, 25122 Brescia (Italy). Tel: 03032988825. Fax: 03029888837. E-mail: rosel.la.levaggi@unibs.it.
†University of Brescia, Department of Economics, Via S. Faustino, 7/b, 25122 Brescia (Italy). E-mail: francesco.menonci@unibs.it
1 Introduction

Tax evasion is a general and persistent problem with serious economic consequences. It affects transition economies as well as countries with developed tax systems. According to recent estimates, the level of tax evasion in Europe is about 20 percent of GDP, accounting for a potential loss of about 1 trillion Euros each year (Murphy, 2014). In countries such as the UK the tax gap is about 6.8 of the total revenue, but in Italy it soars to 180 billion per year (HMR, 2014; Murphy, 2014). The analysis of tax evasion in the tax compliance literature dates back to the classic papers by Allingham and Sandmo (1972); Srinivasan (1973); Yitzhaki (1974) Since then, a large amount of literature relating to corruption and tax evasion has emerged, several solutions have been proposed to tackle this problem, but very little has been achieved. The present economic crisis has put further stress on public accounts and tax evasion is again at the centre of the policy arena.

National Governments try to reduce tax evasion by making audits more effective and to this purpose they use observable and measurable characteristics of taxpayers to target auditing. Recently, predictive analytic tools have become popular (Cleary, 2011; Hashimzade et al., 2014; Hsu et al., 2015). Conspicuous consumption and other individual characteristics are usually used to estimate the actual income and the ensuing gap with what declared. When the difference is “too high”, an audit procedure starts. Most countries keep audit algorithms hidden; in Italy they started to be announced to the taxpayers before they are asked to fill their tax returns since . For self employed and small businesses, the so called “Studi di Settore” determines a cut-off policy: once the government chooses and announces the estimated income, it is common knowledge that all those taxpayers who declare less than this estimation are surely audited. This process has been extended to consumption through the so called “redditometro” (income meter) to gauge whether the living standards of taxpayers matched their declared income (Pedone, 2009).

In spite of these innovations in audit procedures, most of the theoretical literature still models audit as a process that does not depend on individual spending patterns. In our paper we partially bridge this gap by studying tax evasion in a dynamic context where audits are correlated to the level of conspicuous consumption. Our approach stems from the use of conspicuous consumption in presumptive income determination (Logue and Vettori, 2011; Pulina, 2011). In this context, the potential tax evaser must carefully choose how to split his consumption between conspicuous and non conspicuous good in order to reduce the probability of being audited. Two main consequences arise: on the one hand the individual may have less incentive to evade taxes because in the short run the additional income cannot be used to increase conspicuous consumption and, on the other hand, audit on presumptive consumption creates an inter-temporal substitution effect in consumption decisions (Deaton, 1992). In order to capture exceptions are Yaniv (2013) and the literature on the effects of past audits on present tax evasion (Engel and Hines, 1999).
this inter-temporal effect, we model tax evasion decisions for a representative consumer in a dynamic context.

The individual has an initial stock of capital that increases over time following an uncertain path. He obtains utility from consumption of two goods: a normal good and a conspicuous good. Taxes are levied only on income, while consumption is tax free. In each period, the agent may decide to hide part of his income from the tax authorities; in the event of an audit, any detected evasion will lead to the payment of a fine. The most important novelty of our model is that audit is a positive function of the difference between a given percentage (α) of declared income and consumption on conspicuous goods. The parameter α measures the percentage of (declared) income that the Government assumes to be “reasonable” for an agent to consume in conspicuous consumption. Thus, the greater the difference between presumptive and actual conspicuous consumption, the more likely is an audit.

This assumption has a pervasive effect on consumers’ behaviour. In fact, we show that the representative consumer, given the parameters of the audit function, optimally chooses how much to evade and consume and thus indirectly determines the frequency of audit. Our model confirms the Allingham and Sandmo (1972) traditional result that an increase in the tax rate increases tax evasion; what is more relevant is that we find the same result also when the fine is proportional to the tax evaded, contrary to Yitzhaki (1974).

The paper is organised as follows: in Sections 2 and 3 we describe the model; in Section 4 we present the main results, while in Section 5 we show the effect on tax evasion of a change in the fiscal parameters. Finally, Section 5 concludes.

2 Capital accumulation

In a given period of time (a year) the capital $k(t)$ produces income $y(t)$ through the following linear production function:

$$y_t = Ak_t$$

where $A$ is the technology (constant) parameter and measures the total factor productivity (TFP). It is reasonable to assume $0 < A < 1$ since a given amount of capital cannot produce a yield higher than the capital itself. $y_t$ can be saved (and it will produce new capital) or consumed either in the normal good ($g_t$) or in the conspicuous good ($c_t$). Total income is the tax base.

Government levies a tax proportional to income at rate $\tau \in [0, 1]$ which reduces capital accumulation. If the consumer fully reports his income his net accumulation path is

$$dk_t = ((1 - \tau) y_t - c_t - g_t) dt.$$

Capital accumulation may be improved if the agent hides a fraction $e_t$ of his income $y_t$. Nevertheless, if evasion is detected, the agent must pay a fee
(\eta(\tau)) proportional to evaded income and whose level may depend on the tax rate itself:

\[ \eta(\tau) c_t y_t. \]

As in Levaggi and Menoncin (2013, 2012); Bernasconi et al. (2015), the specification of \( \eta(\tau) \) allows to consider several fine regimes:

- \( \eta(\tau) = \eta_0 \): the fine is proportional to evaded income as in Allingham and Sandmo (1972);
- \( \eta(\tau) = \eta_1 \tau \): the fine proportional to evaded tax as in Yitzhaki (1974);
- \( \eta(\tau) = \eta_0 + \eta_1 \tau \): the fine is a combination of these two previous cases.

Evasion adds risk in equation (2) since the fee \( \eta(\tau) \) may or may not be paid. When a fraction \( e_t \) of yield is evaded, the dynamic equation of gross capital accumulation becomes

\[ dk_t = ((1 - \tau + \tau e_t) y_t - c_t - g_t) dt - \eta e_t y_t d\Pi_t, \tag{3} \]

where \( \Pi(t) \) is a jump Poisson process\(^2\) whose (constant) intensity is \( \lambda \in [0, +\infty) \) (and thus the expected value of \( \Pi(t) \) is \( \lambda t \)). The latter measures the frequency of control; when it is zero the probability to be audited is zero, while when it tends towards infinity the probability to be audited tends to 1. The same stochastic process for describing the fee payment is used in Levaggi and Menoncin (2012, 2013); Bernasconi et al. (2015); Levaggi and Menoncin (2015a).

Audits are targeted, i.e. the probability of being audited is not a random event completely outside the control of the representative individual. In particular, we assume that the frequency of the audit (\( \lambda_t \)) depends on the distance between a percentage (\( \alpha \)) of declared income and actual conspicuous consumption. The parameter \( \alpha \) measures the percentage of income that a Government assumes an agent should buy. Thus, the higher this difference the less likely to be audited; in particular, given two positive constant \( \lambda_0 \) and \( \lambda_1 \), we assume

\[ \lambda_t(c_t, e_t) = \lambda_0 - \lambda_1 (\alpha (1 - e_t) y_t (1 - \tau) - c_t). \tag{4} \]

Central Government observes \( e_t, g_t \) and \((1 - \tau)(1 - e_t) y_t \). The latter is used to determine the presumed expenditure in conspicuous consumption (\( \tilde{c}_t \)). The underlying assumption is that consumption (especially of luxury goods) can be used as an income proxy. In other words, conspicuous consumption represents an income meter that allows to determine the presumptive income of the taxpayer. This process is done by setting \( \alpha \) such that \( \tilde{c}_t = \alpha (1 - e_t) y_t (1 - \tau) \). Government finally computes the difference \( \tilde{c}_t - c_t \).

Parameters \( \lambda_0 \) and \( \lambda_1 \) play different roles. The former is a measure of the idiosyncratic risk of being caught since it represents the frequency of audit that is outside the control of the taxpayer. Instead, the latter is a measure of the sensitivity of controls to the difference between actual and expected conspicuous consumption.

\(^2\)For details about Poisson processes used in finance see Cont and Tankov (2004) and Øksendal and Sulem (2007).
3 Consumer’s preferences

The representative consumer allocates his income among consumption and savings in order to maximise his expected inter-temporal utility over an infinite time horizon. As in Yaniv (2013) utility is defined over two goods: conspicuous consumption ($c_t$) and non-conspicuous consumption ($g_t$).

The idea behind this distinction is that the purchase of some goods can be associated with a specific lifestyle and can be used by consumers to signal their social status to their peers. The basket of conspicuous goods may be more or less wide and it may vary between countries. We assume an additive utility function in the two goods. Given their nature, we will assume risk neutrality in conspicuous consumption as in Srinivasan (1973); Sproule et al. (1980). On the other hand, for “ordinary” consumption, the investor is assumed to be risk averse, with a given amount of subsistence consumption ($\bar{g}$). The (constant) subjective discount rate is $\rho$ and the optimisation problem can be written as follows:

$$\min_{c_t, g_t} \max_{c_t, g_t} E_0 \left[ \int_0^\infty \left( c_t + \beta \frac{(g_t - \bar{g})^{1-\delta}}{1-\delta} \right) e^{-\rho t} dt \right] ,$$

(5)
given the dynamic capital accumulation (3).

The parameter $\beta$ measures how much the normal good is preferred with respect to the conspicuous one as in Yaniv (2013). However, since the probability of being audited is increasing in $c_t$, the agent will try to reduce at minimum his conspicuous consumption.

4 Optimal consumption and tax evasion

In this section we present the optimal consumption and tax evasion path for the representative individual that solves problem (5). The main results are summarised in the following proposition:

**Proposition 1.** The optimal evasion, conspicuous consumption and non-conspicuous consumption solving problem (5), given (1) and (3), are

$$c^*_t = \frac{1}{\lambda_1 \eta_0 y_t} \left( \frac{\rho}{A (1 - \tau)} - 1 \right) ,$$

(6)

$$c^*_t = \alpha (1 - \tau) y_t - \frac{2 (1 - \tau) \left( A^{\frac{\rho}{1-\tau}} - 1 \right) + \eta \lambda_0 - \tau}{A A^{\frac{\rho}{1-\tau}} \eta \lambda_1} ,$$

(7)

$$g^*_t = \bar{g} + \left( \beta \frac{\rho - (1 - \alpha) (1 - \tau) A^{\frac{\rho}{1-\tau}}}{\alpha (1 - \tau) A} \right)^\frac{1}{2} .$$

(8)

Proof. See Appendix (A).

\□
The model has an interior solution only if the following conditions are fulfilled:

\[
\begin{align*}
\rho & \in (A (1 - \tau), A), \\
\tau & \in (0, 1 - \frac{1}{1 + \lambda_1 \eta y} \frac{\rho}{A}).
\end{align*}
\]  

(9)

If the first condition is not met, evasion becomes negative, and accordingly there is a corner solution at \( e^*_t = 0 \). If the discount rate is higher than the productivity of capital (i.e. the growth rate of capital is lower than the subjective discount rate) there is no scope for saving for the future and also tax evasion is not convenient. However, the net productivity of capital should be lower than the discount rate to have an interior solution which implies that the tax rate should be higher than \( \tau > 1 - \frac{\rho}{A} \). The economic intuition behind this expression is quite interesting from a policy point of view. If the tax rate is “sufficiently low” consumers do not evade. However, when \( \tau \) increases beyond a specific threshold \((1 - \frac{\rho}{A})\) consumers start evading because the tax is so high that it undermines capital accumulation over time. This threshold depends on the productivity parameter \((A)\), and the subjective discount rate \((\rho)\). In particular, the threshold for the tax rate is directly correlated to \( A \), which means that the lower the productivity the lower the tax to avoid tax evasion.

This result is in line with the findings of the recent literature that incorporates social norm shame and stigma in tax evasion models, as in Alm (2012); Calvet Christian and Alm (2014). However, it should be pointed out that our result does not depend on the presence of a social norm which induces individual to pay what they think is a fair amount of taxes. In our model the “fair” level of taxation for which consumers would comply simply depends on economic parameters relating to the “affordability” of the tax burden.

In the tax rate goes above the threshold in the second line of (9), the consumer optimally evades all his income. This upper threshold depends on the same parameters described above, but also on the audit parameters \( \lambda_1, \eta \) and \( \alpha \) which share the effect of increasing the level of the tax rate for which income is fully evaded. If both conditions in (9) are taken into account, we obtain that there must be both a lower and an upper threshold for the tax rate to have an interior solution:

\[
1 - \frac{\rho}{A} < \tau < 1 - \frac{1}{1 + \lambda_1 \eta y} \frac{\rho}{A}
\]

(10)

The optimal solutions (6), (7) and ((8)) allow us to conclude that the representative agent does not want to avoid controls, in fact he wants to optimise their frequency in order to reduce their negative effects. To show this we can substitute equations (6) and (7) into (4) to find the optimal frequency of controls that the consumer is willing (and expecting) to bear:

\[
\lambda_t (e^*_t, e^*_t) = \frac{A - \rho}{\eta A}.
\]

(11)

From equation (9) \( A > \rho \) and hence \( \lambda_t \) is always strictly positive, i.e. the consumers does not try to avoid controls (which would imply \( \lambda_t = 0 \)). In other
words, in equilibrium, the taxpayer optimally chooses the frequency of audit that maximises his intertemporal utility. Given his intertemporal impatience \( \rho \), the representative consumer adjusts to the fiscal parameter \( \eta \) set by the Government and to the economic condition \( A \) in order to reach his optimal expected number of audits.

The percentage of income that the Government assumes should be used to buy conspicuous goods \( \alpha \) affect all the choice variables (6), (7) and (8). An increase in \( \alpha \), for example, increases the conspicuous consumption and reduces tax evasion because the consumer, faced with a reduced probability of being caught increases conspicuous consumption up to the point where \( \lambda \) is again in equilibrium. For the same reason, an “impatient” consumer, i.e. an individual with a relatively high \( \rho \) evades more (see equation 6); if caught he will have to pay a bigger fine, so he has to reduce the probability of being caught, and in fact from equation (11) we can note that there is a negative relationship between \( \lambda_t \) and \( \rho \). To achieve this result the consumer reduces \( c_t^* \) and increases \( g_t^* \) (see proposition 1).

Since 11 is independent of time, the amount of income evaded \( c_t^* y_t \) is over time (see equation 6). However, as the income on average increases, the fraction evaded decreases. This is in line with what expected. In a model like the one presented here, tax evasion allows to increase consumption, especially conspicuous one; however if this is done immediately, the risk of being audited is very high while the consumer wants to keep \( \lambda_t \) at its optimal (constant) level. The best strategy for the consumer is to initially accumulate income that will be later used to increase conspicuous consumption (and declared income in order to keep \( \lambda_t \) constant); in other words there is a limited effect of deferred taxation. In the long run if the revenue derived from the accumulation of the capital that has been evaded is sufficiently high the consumer may pay more than what he would have paid without tax evasion, i.e. the individual may share a part of the profit deriving from investing income that should have been taxed with the tax authority.

Finally, it should be noted that the optimal amount the consumer allocates to non-conspicuous consumption, \( g_t^* \), does not depend on the fine \( \eta \), and the only relevant fiscal parameter is \( \alpha \). The interpretation of this result is again that in setting his non-conspicuous consumption the consumers is trying to control the probability of an audit, independently of the consequences (in terms of fine) of the audit itself.

5 The effects of fiscal policy on optimal tax evasion

Let us study the effect of fiscal parameters on the optimal tax evasion. The elasticity of tax evasion to the tax rate has been one of the most studied and controversial effects in this literature. For the general case, the results of our model are summarised in the following proposition.
Proposition 2. The elasticity of optimal evasion with respect to tax is positive (negative) if and only if the elasticity of the fee with respect to tax is lower (higher) than a given threshold:

$$\frac{\partial e^*_t}{\partial\tau e^*_t} \gtrless 0 \iff \frac{\partial\eta(\tau)}{\partial\tau\eta(\tau)} \gtrless \frac{\rho}{A(1-\tau)} - 1 \gtrless \frac{\tau}{1-\tau}. $$

Proof. It is sufficient to compute the derivative of $e^*_t$ with respect to $\tau$, recalling that $\eta(\tau)$ is a function of $\tau$:

$$\frac{\partial e^*_t}{\partial\tau e^*_t} = \frac{\rho\tau}{A(1-\tau)^2} - 1 - \frac{\partial\eta(\tau)}{\partial\tau\eta(\tau)} \frac{\tau}{\eta(\tau)}, \quad (12)$$

where, from 9 we know that $\frac{\rho}{A(1-\tau)} - 1 > 0$.

In our model both signs may occur, i.e. tax evasion might increase or decrease as a result of an increase in the tax rate.

As in Section 2 we can consider several special cases

- $\eta(\tau) = \eta_0$: fine is proportional to evaded income as in Allingham and Sandmo (1972). In this case $\frac{\partial\eta(\tau)}{\partial\tau\eta(\tau)} = 0$ and $\frac{\partial e^*_t}{\partial\tau e^*_t} > 0$ (as in Allingham and Sandmo, 1972);

- $\eta(\tau) = \eta_1\tau$: fine is proportional to evaded tax as in Yitzhaki (1974). In this case $\frac{\partial\eta(\tau)}{\partial\tau\eta(\tau)} = 1$ so that we can write $\frac{1-2\tau}{(1-\tau)^2} \leq \frac{4}{\rho}$. $\frac{4}{\rho}$ is positive and greater than one while $\frac{1-2\tau}{(1-\tau)^2}$ is equal to one and then decreases (the derivative with respect to $\tau$ is negative). This allows to conclude that $\frac{\partial e^*_t}{\partial\tau e^*_t} > 0$, i.e. the Yitzhaki (1974)’s paradox is not confirmed in this model. This result is not surprising in our context. Equation 10 shows that the tax rate is an important element in determining tax evasion; this result allows us to determine that a change at the margin has the same effect, even if the fine that will have to be paid increases proportionally (due to the increase in the tax rate).

Let us now study the effects of fines and audit frequency. First of all, it is interesting to note that the fine $\eta$ reduces tax evasion. Controls have the same effect, but only $\lambda_1$ is relevant, as one might expect. To understand this result we should go back to equation (4), where we see that the higher $\lambda_1$ the more effective the control of the consumer over $\lambda_t$.

From a policy point of view, the interesting question is to determine which of the two instruments is more powerful.

Proposition 3. If the fine depends on evaded income and evaded tax, the elasticity of optimal evasion with respect to exogenous frequency in controls ($\lambda_1$) and $\alpha$ is equal to -1 and it is higher than the elasticity to a change in the penalty; for more traditional models (fee either on evaded income or evaded taxes) all the audit parameters have the same effect on tax evasion.
Proof. From equation 6 we obtain \( \frac{\partial e^*}{\partial \lambda_1} \frac{\lambda_1}{e^*} = -1 \) while for the fee we get \( \frac{\partial e^*}{\partial \eta_i} \frac{\eta_i}{e^*} = -\tau_i \eta_i \) \( \frac{\eta_i}{\eta_0 + \eta_1 \tau} < -1 \) for \( i = \{0, 1\} \).

Our findings have several policy implications. The first one is that, contrary to the findings in other static and dynamic models (Bernasconi et al. 2015; Levaggi and Menoucin 2015b, 2012; Sandmo 2005) controls are more efficient than fees in reducing tax evasions. In this model, this result depends on the shape of the penalty function which foresees a combination of fee based on both evaded income and evaded taxes. When we revert to a more traditional formulation, all the audit parameters have exactly the same effect. This is an important peculiarity of the model: in practice a change in any of the audit parameters has the same effect on tax evasion. This happens because the penalty acts directly on equation 11. The increase means that the consumer has to reduce the number of controls which in turn implies that tax evasion should be reduced and all the other decision variables should be adjusted. On the other hand, \( \lambda_1 \) and \( \alpha \) increases the probability of being controlled. The consumer has to adjust all his decision variables to get back to his optimal audit frequency. This implies that the effect on conspicuous and non conspicuous consumption may be sensibly different according to the variable used.

Discussion and conclusions

In the recent past some Governments have tried to tackle tax evasion using more targeted audits that should allow to determine the probability that an individual is misreporting his income from observable information (Hashimzade et al., 2014; Jaramillo, 2003).

In our paper we assume audits to be targeted through the level of consumption in specific goods and services (so-called “conspicuous consumption”) that, by their own nature, may be considered a signal of high income (Logue and Vettori, 2011; Pulina, 2011). In this context the individual should have less incentive to evade taxes because if the additional income is used to increase conspicuous consumption, audit will become more likely.

In our context consumer evades only if the tax rate is “too high”. If Government could keep the tax rate below a given threshold, nobody would evade. If for budget reasons this is not possible, evasion will arise and in this case Government will have to balance all the parameters of the audit function. However, our model shows that the effect of these parameters is more important from an economic rather than a fiscal point of view. The effect of these parameters on tax evasion is (especially at the margin) rather equal; on the other hand these variables have quite a different impact on conspicuous and non conspicuous consumption which in the real world might have pervasive effects on economic growth.

In our model we assume that 1) there is a perfect correspondence between the goods that Government considers to be conspicuous consumption and the basket that enters consumer utility function, and 2) the taxpayer can observe all the parameters of the audit function. A more general model should also consider what happens when these assumptions are relaxed and this will be the objective of our future research.
A Optimization problem

Given the optimization problem (5) and the behaviour of the capital in (3) the corresponding Hamilton-Jacobi-Bellman equation is

\[
0 = \frac{\partial J_t(k_t)}{\partial t} - \rho J_t(k_t) + \frac{\partial J_t(k_t)}{\partial k_t} (1 - \tau) Ak_t + \min_{\epsilon_t} \max_{e_t, g_t} \left[ \epsilon_t + \frac{\beta (g_t - \delta - \gamma)^{1 - \delta}}{1 - \delta} + \frac{\partial J_t(k_t)}{\partial k_t} (\tau e_t Ak_t - \epsilon_t - g_t) \right],
\]

where \( J_t(k_t) e^{-\rho t} \) is the value function, whose boundary (transversality) condition is

\[
\lim_{t \to \infty} J_t(\bullet) e^{-\rho t} = 0 \quad (13)
\]

Given (4), the derivatives of \( \lambda_t(\epsilon_t, e_t) \) w.r.t consumption and evasion are

\[
\frac{\partial \lambda_t(\epsilon_t, e_t)}{\partial \epsilon_t} = \lambda_1,
\]
\[
\frac{\partial \lambda_t(\epsilon_t, e_t)}{\partial e_t} = \lambda_1 \gamma y_t (1 - \tau).
\]

The Jacobian of the optimization problem, with respect to \( \epsilon_t, e_t \) and \( g_t \) is

\[
\nabla_{c,e,g} J = \begin{bmatrix}
1 - \frac{\partial J_t(k_t)}{\partial k_t} + \lambda_1 (J_t(k_t - \eta e_t Ak_t) - J_t(k_t)) \\
\frac{\partial J_t(k_t)}{\partial k_t} \tau Ak_t + \lambda_1 \gamma y_t (1 - \tau) (J_t(k_t - \eta e_t Ak_t) - J_t(k_t)) + \lambda_1 (c_t, e_t) \frac{\partial J_t(k_t - \eta e_t Ak_t)}{\partial e_t} \\
\beta (g_t - \delta - 1)
\end{bmatrix},
\]

and the Hessian matrix is

\[
\mathcal{H}_{c,e,g} J = \begin{bmatrix}
0 & \lambda_1 \gamma y_t (1 - \tau) \frac{\partial J_t(k_t - \eta e_t Ak_t)}{\partial e_t} + \lambda_1 (c_t, e_t) \frac{\partial^2 J_t(k_t - \eta e_t Ak_t)}{\partial e_t^2} & 0 \\
\lambda_1 \gamma y_t (1 - \tau) \frac{\partial J_t(k_t - \eta e_t Ak_t)}{\partial e_t} + \lambda_1 (c_t, e_t) \frac{\partial^2 J_t(k_t - \eta e_t Ak_t)}{\partial e_t^2} & 0 & -\beta \left( g_t - \delta - 1 \right)
\end{bmatrix}
\]

In order to solve these FOCs (\( \nabla_{c,e,g} J = 0 \)), a functional form for the guess function must be guessed. Since the utility function in linear, also the value function should be linear in capital. Thus, we assume\(^3\)

\[
J_t(k_t) = F k_t + G,
\]

where \( F \) and \( G \) are constant that must solve the HJB equation. With this functional form the Jacobian and the Hessian become

\[
\nabla_{c,e,g} J = \begin{bmatrix}
1 - F - \lambda_1 \eta e_t Ak_t F \\
\beta (g_t - \delta - 1 - F)
\end{bmatrix},
\]

\(^3\)Note that this form satisfies the transversality condition.
\[
H_{c,e,g} J = \begin{bmatrix}
0 & -\lambda_1 \eta A_k F & 0 \\
-\lambda_1 \eta A_k F & -\lambda_1 2 \alpha y_t (1 - \tau) \eta A_k F & 0 \\
0 & -\delta \beta (g_t - \bar{g})^{-\delta - 1} & 0 \\
\end{bmatrix},
\]
where, for any positive \( F \), we see that the stationary point is a saddle, which has a maximum for the evasion and the non-conspicuous consumption, and a minimum for the conspicuous consumption.

If this guess value function is substituted into the FOCs, the optimal evasion and consumption are obtained as functions of the constant \( F \):

\[
e_t^* = \frac{1 - F}{F} \frac{1}{\lambda_1 \eta},
\]

\[
c_t^* = \alpha y_t (1 - \tau) + \frac{\tau - \eta \lambda_0 - 2 \alpha (1 - \tau) \frac{1 - F}{F}}{\eta \lambda_1},
\]

\[
g_t^* = \bar{g} + \left( \frac{F}{\beta} \right)^{-\frac{1}{\delta}}.
\]

Now, \( c_t^*, e_t^* \) and \( g_t^* \) are substituted into the HJB equation in order to find \( G \) and \( F \). The HJB becomes

\[
0 = -\rho F k_t + (F + (1 - F) \alpha) (1 - \tau) A_k + \frac{\tau - \eta \lambda_0}{\eta \lambda_1} (1 - F)
\]

\[
- \frac{\alpha (1 - \tau) (1 - F)^2}{\lambda_1 \eta} \frac{1}{F} + \frac{\delta}{1 - \delta} (1 - \beta)^{\frac{1}{\delta}} F^{1 - \frac{1}{\delta}} - F g_t^*,
\]

which can be split into two equations, one which contains \( k_t \) and one which does not:

\[
0 = -\rho F k_t + (F + (1 - F) \alpha) (1 - \tau) A_k,
\]

\[
0 = -\rho G + \frac{\tau - \eta \lambda_0}{\eta \lambda_1} (1 - F) - \frac{\alpha (1 - \tau) (1 - F)^2}{\lambda_1 \eta} \frac{1}{F} + \frac{\delta}{1 - \delta} (1 - \beta)^{\frac{1}{\delta}} F^{1 - \frac{1}{\delta}} - F g_t^*.
\]

from the first equation the value of \( F \) can be found:

\[
\frac{1 - F}{F} = \frac{\rho - (1 - \tau) A}{\alpha (1 - \tau) A},
\]

while the value of \( G \) is obtained from the second equation (but it is immaterial to our purposes). After substituting this value of \((1 - F) / F\) and \( F \) into the FOCs, the optimal values \( c_t^*, e_t^* \) and \( g_t^* \) in Proposition (1) are obtained.

References

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