BEQUEST TAXATION AND EFFICIENT ALLOCATION OF TALENTS

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Abstract

In this paper we analyze intergenerational mobility on education. After a brief empirical analysis of the influence of family background on educational attainment, we present a dynamic model where the decisions concerning education may be financially constrained. Therefore, people who get the higher educational levels are not necessarily the more talented. This “misallocation effect” causes a reduction in the efficiency of the economic system. We show that a proportional bequest taxation, whose yield is redistributed among all “younsg”, increases efficiency.

keywords: education, bequest, talents’ allocation.

JEL: D33, I22, I30, J24
1 Introduction

A large body of empirical findings has shown that a correlation between father/mother and children socio-economic status exists\(^1\). Among the most recent papers, Chevalier et al. (2005), using the Labor Force Survey database, confirm that, in the U.K., parents’ and children’s income and education are highly correlated, with stronger effects of maternal education than paternal and stronger effects on sons than on daughters. Comi (2004), using the LIS database, studied intergenerational mobility in income and education in European countries, finding that Italy is the most “immobile” country in Europe.

The aim of this paper is to check the relationship between intergenerational mobility and allocational efficiency, the latter requiring that higher educational levels are attained by more talented individuals.

Assuming that financial constraints are effective in driving educational decisions (as section 3 seems to show) the theoretical model of section 4 predicts a lower ratio of educated individuals coming from non-educated families than from educated ones and affirms the existence of “misallocated”\(^2\).

One of the main finding of the model is that intergenerational redistribution via bequests taxation is desirable, because, helping children endowed with high ability and coming from poor families to circumvent financial constraints, it increases the average talent of skilled people, generating a better ability allocation and an efficiency gain measured in term of average utility\(^3\).

The paper is organised as follows. Section 2 reports a brief survey concerning the intergenerational persistence in status inequality and the influence of bequest taxation on it. In section 3 we examine the intergenerational transition matrices of Italy, US and UK, trying to study the relationship between the allocation of individuals in unskilled/skilled position, their talent and their family background. In section 4 we present a theoretical model where an efficiency problem in talents’ allocation appears and where both parents’ “bequest” and state redistribution are crucial in determining schooling financing and educational attainment. Section 5 concludes.

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\(^2\) In the model people are assumed to be heterogeneous in their talents so that we refer to “misallocated individuals” as those whose position is dependent on the family: unskilled children of unskilled parents who would have been skilled and skilled children of skilled parents who would have been unskilled if the allocation in the skilled position were not dependent on social class but on individual’s talent alone.

\(^3\) Noteworthy, this results appear only in the case that the fiscal yield coming from bequest taxation is redistributed throughout all the population and it does not appear if it is used to finance education.
2 Brief overview of the literature on intergenerational mobility and bequest

Although there are no doubts about the persistence of status inequality, there is no general agreement on the causal mechanisms behind it. Focusing on differences in schooling decisions, two main theories have been developed and empirically checked by economists:

- the most popular theory, started by Becker and Tomes (1979, 1986), emphasizes the role of “short-run” financial constraints, which make it difficult for low income families to enroll their children in high education levels, even if children show high ability during compulsory school.

- the second hypothesis, recently emphasized by Carneiro and Heckman (2002), gives more importance to “long-run” factors, so that high-status children are, on average, the ones who possess the talent required to take advantage of higher education.

Actually, both “short-run” financial constraints and “long-run” family factors play a role in educational attainment persistence, the latter mainly via cultural influences and “ability” acquired in family environment. Moreover, the genetic transmission of talents from mother to children can play a role. Therefore, “nature” and “nurture” components of parental background are important in determining children’s educational outcomes. “Scholastic ability”, usually measured by grade attainments, comes from different sources, because it is both genetically transmitted (nature) and acquired into the family at early ages: richer and more educated families are better off in assisting children to develop cognitive ability (nurture).

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4Checchi (2005) present a complete survey of education related topics and show some conclusions linked up with our issues.

5Checchi D. (2003), with a cross country analysis, suggests that financial constraints limit access to secondary school. Shea (2000) empirical results are potentially consistent with the hypothesis that credit market imperfections constrain low income households to make suboptimal investments in their children; Krueger (2004) reviews various contributions supporting the view that financial constraints have a significant impact on educational attainment. See also Kane(1994), Ellwood and Kane(2000).

6Carneiro and Heckman (2002): “most of the family income gap in enrollment is due to long-run factors that produce the abilities needed to benefit from participation in college”, however, they found that also “short-run” financial constraints play a (minor) role in socioeconomic inheritance.

Bowles and Gintis (2002) decompose status persistence between generations in various channels, concluding: “wealth, race and schooling are important to the inheritance of economic status, but IQ is not a major contributor and (...) the genetic transmission of IQ is even less important”. Even if the correlation of IQ between parents and children ranges between 0.42 and 0.72 and if a positive relation between cognitive ability and earnings is well documented in economic literature (Bowles, Gintis and Osborne (2002), between others), Bowles and Gintis pointed out that IQ is not a relevant determinant of economic success by itself. Plug and Vijverberg (2003), considering differences in educational attainment between adopted children and children who are their parents’ own offspring, found that it is only to a certain extent that ability is an important factor in explaining the educational attainment, but that about 50% of ability relevant for education is inherited.

There are many theoretical models that consider the influence of bequest taxation on inequality and/or on production and growth. Few attempts have been made investigate the relationship between bequest taxation, financial constraints and allocation of talent. Becker and Tomes (1986) consider financial constraints in education, but they do not contemplate bequest taxation and its consequences, simply emphasizing that income taxes reduce incentives to invest in education. More recently, Grossman and Poutvaara (2005), in a framework with a representative agent and intended bequest, suggested that a small bequest taxation may favor efficiency if parents evaluate children education and bequest leaving as substitute goods.

If financial constraints are relevant in leading to educational attainment and the bequest left to children contributes to determining schooling performance, the investigation of the motives for bequest becomes a crucial point. Four categories of motives are mentioned in economic literature:

- the first is based on the idea of altruistic bequests: parents care about the utility of their children.
- the second refers to exchange-related motives that induce old men to remunerate their children for their care taking with an implicit “promise” of a bequest.

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8 One must be careful with these results, because the dataset used in the analysis reports IQ only for one parent.
10 See Cremer and Pestieau (2003) for a complete taxonomy.
the third considers that, in an uncertain world, accidental bequests should exist because people do not know the date of their death\textsuperscript{13}.

- Lastly, parents may receive utility from the act of giving (joy of giving)\textsuperscript{14}. Formally, this sort of bequests are included in the utility function as a consumption in the last period of life, and the crucial question is whether what matters to the parent is the net or the gross of the amount.

The few empirical studies on this topic found some evidence that bequests are clearly intentional\textsuperscript{15} and Page (2003) added that “there is a significant positive correlation between the amount of gift given and tax rates, especially for older households”. This last evidence suggests that what matters to the donor is the “net” and not the “gross” amount left to children and reduces the relevance of the accidental bequests hypothesis. For our approach is relevant to remember that Hurd and Smith (1999) found that more educated agents are more likely to leave pensions than less educated and that Fink et al. (2005) found that childrens' wealth does not affect parental bequests at all, suggesting that simple models of joy of giving could be the most suitable way to model bequest behaviour.

3 Empirical transitions and abilities

Individual data needed to produce empirical evidence about mobility and abilities are social background (e.g. parents education), educational attainment and a proxi for talent. The last information is particularly difficult to be obtained. The economic literature considers IQ level, results of literacy and mathematical proof made at early ages, or grades obtained at the end of school courses\textsuperscript{16}. All these indicators, referring to students aged usually above 12, are obviously influenced by the ability acquired in families, hence they measure the “scholastic” talent and not the “genetic” one.

Our analysis is based on three different databases: the Italian 2002 SHIW

\textsuperscript{14}Cremer and Pestieau (2003), Glomm and Ravikunar(1992).
\textsuperscript{15}See Bernheim et al. (2001), Joulfaian (2005), Fink et al. (2005).
database\textsuperscript{17}, the British 1999 BCS database \textsuperscript{18}, the US NLSY 1997 database\textsuperscript{19}.

Given that these three databases collect information in a very different way, that they refer to different cohort of individuals (all ages for Italy, people aged around 40 in 2000 for Great Britain and people aged between 17 and 23 in 2002 for the US) and that they present different classification for educational attainment, we will not use them for comparisons between countries\textsuperscript{20}.

Using these databases, we present:

- the two states intergenerational transition matrix (educational attainment of parents and children); section 3.1);

- the quota of \textit{bad allocated} individuals, defined by the ratio of unskilled

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\textsuperscript{17}The 2002 SHIW database is built by the Bank of Italy. Families are the object of the survey. Our individuals are householders and spouses/partners (whose father’s and mother’s education is available from the survey) as well as children living in the family who have stopped studies (4690 individuals). The talent can be proxied for those people who get the \textit{maturità} title (higher secondary school certificate) or more alone, by means of the grade obtained at the end of the educational process (we use the standardised relative grade). Therefore, we consider as unskilled all those individuals whose highest educational level is \textit{maturità}, and as skilled all those individuals who completed university. In order to increase the number of skilled parents, we consider “skilled” all fathers and mothers with the “maturità” or more.

\textsuperscript{18}We analyze the cohort of individuals who answered to propensity scores in 1970, and who were re-interviewed in 1999 (5613 individuals). For these observation we know parents’ education, the result of propensity score at the age of 10 and the highest educational level obtained at the age of around 40. Our categories of skilled/unskilled distinguish between people who obtained the \textit{A-level} (at school until about the age of 16) or more from people who do not get it. As a measure of talent, we consider the \textit{British Score Assessment} (BSA) in verbal method (the sum of \textit{acceptable} answers to “word definition” and “similarities”) and the BSA in quantitative methods (the sum of \textit{acceptable} answers to “recall of digit” and “matrices items”). The data presented in the text refer to the standardized sum of answers of both indicators.

\textsuperscript{19}The NLSY97 consists of a nationally representative sample of approximately 9000 individuals who were aged between 12 and 16 in 1996. Round 1 of the survey took place in 1997. In that round, both the individual and one of her parents were interviewed. Youths are consecutively interviewed every year collecting extensive information about labor market behavior and educational experiences over time. We analyse data from round 5, considering the children educational status in 2002 and defining skilled those who enrolled in college. We use the standardised PIAT score (whose results, corrected by the age of the answerers, are available from 1997 to 2002) as a measure of individual talent; for students who answered the test more than once, we use the earlier score (using the average score does not affect our results). The sample is composed of 4415 individuals.

\textsuperscript{20}Countries can be compared using the TIMSS database that presents scores in Literacy and Math for students aged 14-18 in different countries and information on parents’ education. Unfortunately, educational attainment of students has not been recorded. However, their \textit{future intentions} with respect to their studies have been asked. Obviously, this is a different information from the one considering educational attainment. Therefore we will not use the TIMSS database here.
(skilled) individuals with unskilled (skilled) parents who, given their talent, should have (not) obtained the higher degree.

3.1 Transitions

As emphasised by the economic literature, intergenerational mobility is far from being perfect\textsuperscript{21}. These results are strongly confirmed for Italy (table 1), UK (table 2) and the US (table 3). In fact we always obtain that unskilled families show a lower percentage of skilled children than skilled families.

Table 1: Transition matrix, Italy, 2002 - Individuals with at least the secondary school

<table>
<thead>
<tr>
<th>Parents’ education</th>
<th>Education</th>
<th>Secondary Sch.</th>
<th>University</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below Secondary Sch.</td>
<td>obs</td>
<td>2581</td>
<td>588</td>
<td>3169</td>
</tr>
<tr>
<td></td>
<td>%row</td>
<td>81.45</td>
<td>18.55</td>
<td>100</td>
</tr>
<tr>
<td>Secondary Sch. or above</td>
<td>obs</td>
<td>925</td>
<td>596</td>
<td>1521</td>
</tr>
<tr>
<td></td>
<td>%row</td>
<td>60.82</td>
<td>39.18</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>obs</td>
<td>3506</td>
<td>1184</td>
<td>4690</td>
</tr>
<tr>
<td></td>
<td>%row</td>
<td>74.75</td>
<td>25.25</td>
<td>100</td>
</tr>
</tbody>
</table>

Source: SHIW database

Table 2: Transition matrix, Great Britain, 1999 - cohort of individuals born in 1970

<table>
<thead>
<tr>
<th>Parents’ Education</th>
<th>Education</th>
<th>Below A Level</th>
<th>A level or above</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below A level</td>
<td>obs</td>
<td>3232</td>
<td>1341</td>
<td>4573</td>
</tr>
<tr>
<td></td>
<td>%row</td>
<td>70.68</td>
<td>29.32</td>
<td>100</td>
</tr>
<tr>
<td>A level or above</td>
<td>obs</td>
<td>498</td>
<td>542</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>%row</td>
<td>47.88</td>
<td>52.12</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>obs</td>
<td>3730</td>
<td>1883</td>
<td>5613</td>
</tr>
<tr>
<td></td>
<td>%row</td>
<td>66.45</td>
<td>33.55</td>
<td>100</td>
</tr>
</tbody>
</table>

Source: BCS database

It emerges that the probability of getting education for children of unskilled parents is less than a half of the same probability for children of skilled, both for Italy and the US, whereas in the UK this measure is slightly higher than one half.

\textsuperscript{21}As said in introduction, see i.e. Chevalier et al. (2005), Ermisch and Francesconi (2001).
Table 3: Transition matrix, USA, 2002 - cohort of individuals born between 1980-84

<table>
<thead>
<tr>
<th>Parents’ Education</th>
<th>Below College</th>
<th>College or above</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below College</td>
<td>obs 1655</td>
<td>625</td>
<td>2280</td>
</tr>
<tr>
<td>%row</td>
<td>72.59</td>
<td>27.41</td>
<td>100</td>
</tr>
<tr>
<td>College or above</td>
<td>obs 844</td>
<td>1291</td>
<td>2135</td>
</tr>
<tr>
<td>%row</td>
<td>39.53</td>
<td>60.47</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>obs 2499</td>
<td>1916</td>
<td>4415</td>
</tr>
<tr>
<td>%row</td>
<td>56.60</td>
<td>43.40</td>
<td>100</td>
</tr>
</tbody>
</table>

Source: NLSY97 database

3.2 Misallocated individuals

The availability of data on transitions and some proxies for talent (see notes 17, 18 and 19), allow us to stress if an efficiency problem arises, hence, if there are “misallocated individuals”.

Let us define what we intend by “misallocated individuals”.

We define the probability of being skilled as $q^S$, the probability of being skilled conditional on having unskilled parents as $q^{US}$ and the probability of being skilled conditional on having skilled parents as $q^{SS}$.

Let $G(a)$ be the cumulated distribution of talent in the whole population; $G^U(a)$ and $G^S(a)$ are, respectively, the cumulated distribution of talent conditional on having unskilled parents or skilled parents.

Therefore, $\tilde{a}^S \equiv [G(q^S)]^{-1}$, $\tilde{a}^{US} \equiv [G^U(q^{US})]^{-1}$ and $\tilde{a}^{SS} \equiv [G^S(q^{SS})]^{-1}$ represent the “theoretical” minimum talent required to become skilled for the whole population, for children of unskilled parents and for children of skilled parents, respectively.

We define “misallocated individuals” those people whose ability is such that:

- $\tilde{a}^S < a_i \leq \tilde{a}^{US}$ if parents are unskilled and they are unskilled;
- $\tilde{a}^{SS} < a_i < \tilde{a}^S$ if parents are skilled and they are skilled.

In fact, our “misallocated individuals” are those whose position is dependent on the family:

- unskilled children of unskilled parents who would have been skilled if the allocation in the skilled position was not dependent on social class ($p^{U}_{badA} = Prob[\tilde{a}^S < a_i \leq \tilde{a}^{US}]$);
Table 4: Probability of misallocation (%)

<table>
<thead>
<tr>
<th></th>
<th>SHIW, Italy</th>
<th>BCS, UK</th>
<th>NLSY, USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{badA}^{U}$</td>
<td>3.47</td>
<td>2.25</td>
<td>5.18</td>
</tr>
<tr>
<td>$p_{badA}^{S}$</td>
<td>7.36</td>
<td>7.21</td>
<td>3.19</td>
</tr>
</tbody>
</table>

- skilled children of skilled parents who would have been unskilled if the allocation in the skilled position was not dependent on social class ($p_{badA}^{S} = \text{Prob}[\tilde{a}^{SS} < a_i < \tilde{a}^{S}]$).

Table 4 reports $p_{badA}^{J}$ values, with $J = U, S$, for the three databases, where the talents are measured by means of the grade obtained at some level of the educational process or in some attitude tests (again see notes 17, 18 and 19).

Different measures of talent imply different results in the empirical evidence. We know that the measure used in the previous section is a “scholastic ability”, affected by the talent acquired in family in the early years of life (and by the quality of school and many others factors) that is likely to be greater in an “educated family”.

In order to build a proxy of “innate” talent, or at least purified by every family factors, we estimate our measure of talent on parents educational attainments and we use the residuals to compute an innate ability. Hence, our measure of “inborn” talent is the residual of the following regression:

$$a_i = \beta_0 + \beta_1 \text{Parents}_i + \epsilon$$

where Parents is a dummy indicating the highest degree obtained between father and mother of the individuals.

Table 5 presents the results about “misallocated individuals” calculated using our measure of “innate ability” as the residuals of the previous equation. As expected, the frequency of “bad allocated” individuals is higher for all countries with reference to the frequency calculated in the “scholastic ability case” (table 4).

It is crucial to remember that, probably, the real “genetic ability” is something between the “scholastic ability” (influenced by early years in family) and our measure of “innate ability” (cleaned by all parents effects, even genetic transmission); therefore we can argue that the frequencies of bad allocated individuals should be collocated between the values displayed in tables 4 and 5.\footnote{We are obviously aware that our innate ability does not reflect the true “genetic” ability, because of the genetic transmission of talent, documented by Bowles and Gintis (2002) and others. Our “innate ability” is, in fact, simply the “scholastic ability” constrained to the same average both for children of skilled and unskilled. It may represent “genetic” ability only assuming no genetic transmission of ability between generations.}
Table 5: Probability of misallocation (%): 0 average in ability both for skilled and unskilled (%)

<table>
<thead>
<tr>
<th></th>
<th>SHIW, Italy</th>
<th>BCS, UK</th>
<th>NLSY, USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{badA}^U$</td>
<td>6.69</td>
<td>5.07</td>
<td>12.68</td>
</tr>
<tr>
<td>$p_{badA}^S$</td>
<td>17.62</td>
<td>17.88</td>
<td>13.07</td>
</tr>
</tbody>
</table>

Although we can’t exactly calculate the upgrade of “misallocated individuals”, these empirical results suggest that an efficiency problem actually arises.

4 The model

In this section we present a “non-overlapping” generation model which is able to justify the empirical results concerning talents’ allocation.

We assume that:

- individuals are heterogeneous in their talents;
- genetic talent transmission does not take place (talent is random);
- decisions concerning education are financially constrained, and therefore be depending both on parents’ bequest and government’s redistribution;
- the cost of education is decreasing in talent;
- production is increasing in (or not affected by) talent;
- the decisions concerning education, the amount of bequest to be left to children and the effort on the workplace are the endogenous variables.

We present our model through the following steps: first of all we analyze the individual behavior in order to determine the optimal choice of effort, bequest and educational level (which can be financially constrained), secondly we analyze the intergenerational mobility at family level, that will depend on the “convenience” condition (the preference for getting education) and on the “possibility” condition (the capability of financing it); thirdly, considering the steady state (defined as flows equilibrium), we investigate on the welfare effects of a bequest taxation whose yields are redistributed among all “youngs”. Finally we analyze welfare consequences of bequest taxation in the case yields are used to finance the education system.
4.1 Utility

The economy is composed of a set of agents of unitary mass living for one period and interested both in their consumption and on the bequest they leave to their (unique) child. Each of them must choose:

- her skill level (to get or not a given educational level);
- her effort on the workplace;
- the bequest to leave to her child.

Every individual in the economy is exogenously endowed with a given talent, which we label with $a_{i,t}$, for $0 \leq a_{i,t} \leq 1$, where $i$ indicates the family and $t$ the generation. As explained in the introduction, we assume that the talent of an individual is independent of the talent of her mother.

Effort (endogenous), talent (exogenous) and educational level (endogenous, but financially constrained as we will see later) determine the amount of earnings. We assume that, for given effort and talent, the income of skilled individuals is $\mu > 1$ times the one of unskilled individuals, and we also assume that the cost of education is decreasing in individual talent.

In what follows we use the notation $y_{S,U}^{i,t}$ to indicate the variable $y$ referring to an unskilled individual (second suffix $U$) born from a skilled mother, (first suffix, $S$, so that the first suffix always refers to the mother and the second to the individual). The index $i$ indicates the family and the index $t$ the generation. The notation $y_{S}^{i,t}$ refers to the variable $y$ of an individual born from a skilled mother in case the position of the individual, indicated by the point, is non relevant. The notation $y_{S,U}^{i,t}$, for $J = U, S$ refers to the variable $y$ of an individual born from a skilled mother in the case the position of the individual (which can be both of unskilled or skilled) is relevant.

Consider a skilled individual endowed with talent $a_{i,t}$. We assume that her consumption level is given by:

$$C_{J,S}^{i,t} = \Sigma_{i,t}^{J} - \xi(1-a_{i,t}) + \mu(x_{i,t}^{J,S} + \lambda a_{i,t}) - S_{i,t}^{J,S} \text{ for } J = U, S$$

where:

- $\Sigma_{i,t}^{J} = S_{i,t-1}^{J}(1-T) + E(S_{t-1})T$ for $J = U, S$ is the endowment received by individual $i$ from the previous generation, depending on the amount left

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23Following, for example, Michel and Pestieau (2004) and Glomm and Ravikumar (1992) we consider bequests as a consumption in the last period of life.
to her by her mother ($S_{i,t-1}$) taxed at rate $T$ and on the amount obtained by redistribution ($E(S_{t-1})T$), depending on the average bequest\(^{24}\):

- $\xi(1 - a_{i,t})$ is the cost of education, decreasing in individual talent ($a_{i,t}$);
- $x_{i,t}^{JS}$ is the level of effort which, together with the exogenous talent multiplied by the parameter $\lambda \geq 0$ (which measures the capability of creating income by means of ability), determines the income of individual\(^{25}\);
- $\mu$ is the skill premium;
- $S_{i,t}^{JS}$ is the amount she decides to leave to her child.

The consumption level of an unskilled individual is therefore given by:

$$C_{i,t}^{JU} = \Sigma_{i,t}^{JU} + (x_{i,t}^{JU} + \lambda a_{i,t}) - S_{i,t}^{JU} \quad \text{for} \quad J = U, S$$

because she does not spend on education and, for given talent and effort, perceives an income $\frac{1}{\mu}$ times lower than the one of skilled individual.

We assume that utility of a generic individual, other than on consumption and effort, may also depend on the amount of income not consumed during her life but left to her child (our “bequest”). Let us define as “psychological” utility the utility deriving from bequest.

In the case of *altruistic bequests*, mother’s “psychological” utility should depend both on the amount of her bequest net of taxes and on the amount her child receives by means of redistribution. In the *exchange related motives* and in the case of *joy of giving* “psychological” utility will probably depend more on the gross amount of bequest than on the net amount. Finally, in the case of *accidental bequests*, mother’s “psychological” utility surely depends on the gross amount of bequest, because in that case mothers are not interested in welfare of their children.

Therefore, “psychological” utility ($UP_{i,t}^{J}$) is influenced negatively by taxes. We assume that it can be written as follow:

$$UP_{i,t}^{J} = \rho f(S_{i,t}^{J}(1 - gT)) \quad \text{for} \quad J = U, S$$

where $\rho$ is a parameter reflecting altruism toward children (or a parameter depending on risk aversion in the case of accidental bequests) , $S_{i,t}^{J}$ is the gross

\(^{24}\)In our model, the fiscal share $T$ hits all the wealth given in life and left by parents to their children. Actually, only taxes on gifts and bequests exist, so our $T$ can not be interpreted as the actual tax rate on bequests, but should be strongly lower than it.

\(^{25}\)Therefore, $\mu(x_{i,t}^{JS} + \lambda a_{i,t}$ is the total revenue produced by the individual. Assuming separability between effort ($x_{i,t}^{JS}$) and talent ($a_{i,t}$) implies that effort creates revenue independently on talent and viceversa.
amount of bequest left by a skilled ($J = S$) or unskilled ($J = U$) mother, and $g$ a parameter which signal the relative importance of the different motives for bequest. For instance, if $g = 1$ every increase in the tax rate reduces utility (the case of altruistic bequest)\textsuperscript{26} If $g = 0$, increase of taxation does not affect “psychological utility”. This represents the pure case of accidental bequest, and it may probably approximate the exchange related motives and the joy of giving motives. In what follow our results will be analyzed for the different value of $0 \leq g \leq 1$; nevertheless, we think that a value for $g$ close to zero is a good approximation of the real world.

For all individuals we assume a semi-linear utility function separable in consumption, effort and on the “psychological utility”:

$$U_{i,t}^J = f(C_{i,t}^J) - \gamma x_{i,t}^J + \rho f(S_{i,t}^J (1 - gT)) \quad \text{for} \quad J = U, S$$

where $\gamma$ is a measure of the disutility of effort and $C_{i,t}^J$ and $C_{i,t}^U$ are defined respectively in equations 1 and 2.

Differentiating equation 3 with respect to $S_{i,t}^J$ and using the definition of $C_{i,t}^J$ (equations 1 and 2), it emerges that $f'(C_{i,t}^J) = \rho f'[S_{i,t}^J (1 - gt)]$.

Let us assume that the utility function of equation 3 is logarithmic in consumption and bequest\textsuperscript{27}. In that case $C_{i,t}^J = \frac{1}{\rho} S_{i,t}^J$ must hold for $J = U, S$; individuals allocate their overall income so that bequest is a share $\rho$ of consumption in the whole life-cycle.

Therefore, if an individual chooses to acquire education her utility is:

$$U_{i,t}^{JS} = \ln[\sum J_i - \xi(1 - a_{i,t}) + \mu(x_{i,t}^JS + \lambda a_{i,t}) - S_{i,t}^{JS}] - \gamma x_{i,t}^{JS} + \rho \ln(S_{i,t}^{JS} (1 - gT))$$

whereas if she remains unskilled, she gets:

$$U_{i,t}^{JU} = \ln[\sum J_i + (x_{i,t}^{JU} + \lambda a_{i,t}) - S_{i,t}^{JU}] - \gamma x_{i,t}^{JU} + \rho \ln(S_{i,t}^{JU} (1 - gT))$$

\textsuperscript{26}In that case we should consider into the “psychological utility” also the amount received by children through redistribution. Doing so, the model becomes immediately intractable. Therefore, not considering redistribution into the “psychological utility”, we reduce the “psychological disutility” created by bequest taxation and redistribution. Our results will be biased downward, in the sense that the effect of a bequest taxation on individual utility will be better than the ones obtained throughout our model.

\textsuperscript{27}Using a semi-linear utility function we are able to obtain analytical results for endogenous variables and check for the efficiency of bequest taxation. This specification of the utility function makes the amount of bequest not dependent on the ability, but simply on the status of the individual (skilled/unskilled). Without this simplification, even if it is possible to solve for the convenience condition and the possibility condition (see below), results are very hard to be manipulated in terms of conditional probabilities. The main results of the model, and in particular the “allocation effect” which we will introduce later, do not depend on this specification.
First order conditions of equations 4 and 5 give the optimal choice level for effort $x_{i,t}^J$ and bequest $S_{i,t}^J$, respectively in the case the individual gets/does not get the skilled position:

$$S_{i,t}^S = \frac{\rho \mu}{\gamma}$$  \hspace{1cm} (6)

$$x_{i,t}^J = \frac{1 + \rho}{\gamma} - \frac{\Sigma_{i,t}^J + (\xi + \mu \lambda) a_{i,t} - \xi}{\mu} \quad \text{for} \quad J = U, S$$  \hspace{1cm} (7)

$$S_{i,t}^U = \frac{\rho}{\gamma}$$  \hspace{1cm} (8)

$$x_{i,t}^J = \frac{1 + \rho}{\gamma} - \left[ \Sigma_{i,t}^J + \lambda a_{i,t} \right] \quad \text{for} \quad J = U, S$$  \hspace{1cm} (9)

where the bequest does not depend on talent and on taxation, and it is differentiated among individuals only because of their educational attainment. The amount left to children is not dependent on $T$ because of the logarithmic form of the utility function: this result seems to be coherent with empirical data, where the relationship between tax rate and average bequest has not clear sign\textsuperscript{28}.

**Proposition 1** Definition of endogenous variables

*Bequest and consumption depend on preference parameters only.*

*Effort is decreasing in talent and in the endowment received from the previous generation.*

*Individual whose mother left an amount lower than the average bequest should produce a lower effort if the tax rate increases (and viceversa).*

*If an individual gets a skilled position she produces more effort and she leaves more money to her child.*

**Proof**: see appendix.

Substituting these optimal values in equations 4 and 5, we obtain that indirect utility functions are given by:

$$U_{i,t}^{JS} = \rho \ln(\rho(1 - gT)) - (1 + \rho) \ln \left( \frac{\gamma}{\mu} \right) - \gamma x_{i,t}^{JS} \quad \text{for} \quad J = U, S$$  \hspace{1cm} (10)

$$U_{i,t}^{JU} = \rho \ln(\rho(1 - gT)) - (1 + \rho) \ln(\gamma) - \gamma x_{i,t}^{JU} \quad \text{for} \quad J = U, S$$  \hspace{1cm} (11)

where $x_{i,t}^J$, for $J = U, S$, is defined in equations 7 and 9.

\textsuperscript{28}Cremer and Pestieau (2003).
4.2 Getting education

In this economy, people will get the skilled position spending on education if the following two constraints are filled:

1. the \textit{convenience} condition, so that the indirect utility of being in the skilled position is higher than the indirect utility of being in the unskilled one: \[ U_{i,t}^{JS} > U_{i,t}^{JU}, \text{ for } J = U, S; \]

2. the \textit{possibility} condition, depending on the assumption of imperfect capital market: given their talent, only individuals who receive “enough” money from the previous generation can get the skilled position: \( \Sigma_{i,t}^{J} > \xi(1 - a_{i}) \).

Both conditions depend on the mother’s qualification and on the individual’s talent. To simplify notation, we define:

\[ m \equiv \frac{\xi}{\rho \mu} \equiv \frac{\xi}{\rho_{S}} > 1 \]

as the ratio between education costs of the worst skilled individual (the one with talent 0) and the bequest left to children by skilled individuals\(^{29}\).

Let us start with the \textit{convenience} condition of point 1. The difference between equations 10 and 11 gives:

\[ U_{i,t}^{JS} - U_{i,t}^{JU} = (1 + \rho) \ln(\mu) - \gamma(x_{i,t}^{JS} - x_{i,t}^{JU}) \quad \text{for } J = U, S \quad (12) \]

Note that, according to the 7 and 9, the sign of \( x_{i,t}^{JS} - x_{i,t}^{JU} \) depends on mother position and on individual ability.

\textbf{Proposition 2} \textit{The convenience condition}

\textit{An individual finds it convenient to get education if her talent is higher than a given threshold, which depends on the position of her mother: } \( a_{i,t} > a_{i}^{U} \), for \( J = U, S \).

\textit{The convenience threshold is lower for children of unskilled mothers } \( a_{i}^{U} < a_{i}^{S} \), unless \( T = 1 \).

\textit{Bequest taxation increases the convenience threshold for children of unskilled mothers and reduces the one for children of skilled ones.}

\textbf{Proof:} see appendix.

But not all individuals can get educated: again, the \textit{possibility} constraint of the previous point 2 may be fulfilled or not according to the position of the mother and the ability of the individual.

\(^{29}\)We assume that \( m \) is higher than 1, so that at least the less talented children of skilled mothers are financially constrained.
Proposition 3 The possibility condition

An individual finds it possible to get education if her talent is higher than a given threshold, which depends on the position of her mother: \( a_{i,t} > \tilde{a}_{t}^{J} \) for \( J = U, S \).

The possibility threshold is always higher for children of unskilled mothers \( (\tilde{a}_{t}^{S} < \tilde{a}_{t}^{U}) \), unless \( T = 1 \).

Bequest taxation reduces the possibility threshold for children of unskilled mothers and increases it for children of skilled ones.

Proof: see appendix.

Which of the two constraints is more stringent? Some of the children of unskilled/skilled mother would like to get education but they can not or they can get educated but they do not want to? To answer this question we must compare the constraints previously indicated.

Suppose \( \tilde{a}_{t}^{S} > \tilde{a}_{t}^{U} \), so that children of skilled mother whose talent is such that: \( \tilde{a}_{t}^{S} \leq a_{i,t}^{S} < \tilde{a}_{t}^{S} \) would like to get education but they can not. If this inequality holds, we easily obtain (see propositions 2 and 3): \( \tilde{a}_{t}^{U} > \tilde{a}_{t}^{S} > \tilde{a}_{t}^{S} > \tilde{a}_{t}^{U} \), so that all individuals are constrained solely by the possibility constraint.

Given the analytical definition for \( \tilde{a}_{t}^{J} \) and \( \tilde{a}_{t}^{J} \) (for \( J = S, U \), see equations, ii, iii, v, iv in the appendix) and calculating the above inequality for \( T = 0 \), after some algebraic steps, it is possible to show that:

\[
\tilde{a}_{t}^{S} > \tilde{a}_{t}^{S} \quad \text{if} \quad \frac{\rho}{1 + \rho} < \frac{\ln(\mu)}{\mu} \tag{13}
\]

which may be fulfilled or not according to the values of the parameters. For \( \rho > \frac{1}{1 + e} = 0.582 \) it is never fulfilled. For lower values of \( \rho \) the previous condition can be respected according to the values of \( \mu \). For example\(^{31}\), if \( \rho = \frac{1}{2} \) it is respected if \( 1.85 < \mu < 4.5 \), if \( \rho = \frac{1}{3} \), it is respected for \( 1.29 < \mu < 12.71 \). In the following part of the paper, we make the following hypothesis:

Assumption 1 The parameters of the model are such that condition 13 is always respected, so that some of the individuals are financially constrained (they would like to get education but they can not) whereas none of them can get education but does not want to get it.

\(^{30}\)Given that \( \tilde{a}_{t}^{S} \) is increasing in \( T \) whereas \( \tilde{a}_{t}^{S} \) is decreasing, the above inequality is more stringent in the case of \( T = 0 \).

\(^{31}\)Be aware that \( \rho \) represents the ratio between bequest and consumption in the whole life-cycle. Low values of \( \rho \) seem to be a “reasonable approximation” of the real world.
4.3 The steady state

The next step is the definition of $\phi$, the ratio of skilled workers in the whole population.\(^{32}\)

The evolution of the skilled ratio follows:

$$\phi_t = \phi_{t-1} - \phi_{t-1} p_{SU}^t + (1 - \phi_{t-1}) p_{US}^t$$

Being $G$ the cumulate distribution of talent, $p_{SU}^t = G(\tilde{a}^S_t)$ indicates the probability that a child of a skilled mother becomes unskilled and $p_{US}^t = 1 - G(\tilde{a}^U_t)$ is the probability that a child of an unskilled mother becomes skilled.

In steady state flows between skilled and unskilled individuals are such that the ratio of skilled individuals in the population remains constant over time. Therefore, unless necessary, we will drop the index $t$. The flow condition: $\phi p_{SU} = (1 - \phi) p_{US}$ must hold, defining a constant skilled ratio:

$$\phi = \frac{p_{US}}{p_{US} + p_{SU}} \quad (14)$$

Let us now assume that the talent of individuals is uniformly distributed between 0 and 1. Given this hypothesis and the definition of $\tilde{a}^J$, it is easy to calculate:

$$p_{UU} = \tilde{a}^U, \quad p_{US} = 1 - \tilde{a}^U, \quad p_{SU} = \tilde{a}^S, \quad p_{SS} = 1 - \tilde{a}^S \quad (15)$$

(see figure 1). Solving the system of equations\(^{33}\) 14 and 15 together with the definitions of $\tilde{a}^S$ and $\tilde{a}^U$, shown in the appendix (equations v and iv), we obtain the steady state skill ratio:

$$\phi^*(m, \mu) = \frac{1}{\mu (m - 1) + 1} \quad (16)$$

\(^{32}\)We assume that the offer of skill creates its own demand. This happens both in the self-employment case or assuming a linear production function of the type $y = \mu \phi + (1 - \phi)$ where $y$ is output. See also note 34.

\(^{33}\)The same result is obviously obtained calculating the ergodic of the transition matrix defined by the probabilities $p^J$.

\(^{34}\)The premium for skilled individuals, $\mu$, should depend on the ratio of skilled workers in the population, so that $\mu = \mu(\phi)$. For instance, suppose that aggregate production function is $y = [\zeta \phi^I + (1 - \phi)^I]^r$. In that case, given $\mu$ the ratio between the cost of skilled and the cost of unskilled workers (the latter normalized to the unity), it is easy to obtain the demand for skilled individuals: $\phi^D = \left[1 + \frac{\mu}{\zeta (m - 1)} \frac{1}{\mu} \right]$. Given the supply of skill defined by equation 16, the equilibrium is described by: $\phi^* = \left[1 + \zeta (m - 1) \frac{1}{\mu} \right]^{-1}$ and $\mu = \zeta \phi^I (m - 1) \frac{1}{\mu}$ so that both $\mu$ and $\phi$ depend on the parameters of the production function and on the value of $m$. Given that we are mostly interested in the effect of bequest taxation on aggregate utility and given that $\phi$ is not dependent on the tax rate, in this version of the paper we prefer to assume that $\nu = r = 1$ and $\zeta = \mu$ obtaining the result of the text.
Figure 1: The effects of bequest taxation

Substituting $\phi^*(m, \mu)$ in equations v and iv in the appendix we can solve for the constant probability of change of state (from skilled to unskilled and vice versa) between sequential generations of the same family, and finally we can write the steady state transition probabilities of equation 15 in an explicit form:

\[ p^{US} = \left[ 1 - \left( 1 - \frac{1}{\mu} \right) \frac{1 - T}{m} \right] \phi^*(m, \mu) \] (17)

\[ p^{SU} = \mu(m - 1) \left[ 1 - \left( 1 - \frac{1}{\mu} \right) \frac{1 - T}{m} \right] \phi^*(m, \mu) \] (18)

From equations 17 and 18 we obtain the following result:

**Proposition 4** The allocation effect

The probability that a child of an unskilled mother can achieve a skilled position and the probability that a child of a skilled mother becomes unskilled are increasing in $T$ (see figure 1). Intergenerational educational mobility is increasing in $T$.

*Proof*: see appendix.

4.4 Bequest taxation and individual utility

What are the effects of bequest taxation on individual utility? From equation 10 and 11, we obtain that utility is affected by bequest taxation throughout effort and the fiscal charging, which reduces the amount received by children (J=U,S):

\[ \frac{dU_i^{JS}}{dT} = -\rho \frac{g}{1 - gT} - \gamma \frac{dx_i^{JS}}{dT} \]

\[ \frac{dU_i^{JU}}{dT} = -\rho \frac{g}{1 - gT} - \gamma \frac{dx_i^{JU}}{dT} \]
Therefore, bequest taxation affects individual utility through two different channels: the first, negative or nil (if $g = 0$) depends on the reduction of the “psychological utility”; the second depends on variation on effort; it is positive for children of unskilled and negative for children of skilled (see the proof of the next lemma).

**Proposition 5** Individual utility

*Bequest taxation reduces utility of children of skilled mothers. It can raise utility of children of unskilled mothers (depending on the values of the parameter $g$) and, for some of them, it relaxes the possibility constraint.*

**Proof:** see appendix.

In fact, (see equations 10 and 11) when taxation increases:

- Children of skilled mothers will produce an higher effort and they obtain a lower “psychological utility” (unless $g = 0$), so that, for all of them, utility decreases. Furthermore, for some of them the possibility constraint strengthens and they can not get education,

- Children of unskilled mothers will produce a lower effort but they obtain a lower “psychological utility” (unless $g = 0$). For $g$ not to high, the utility of children of unskilled parents is increasing in $T$. Furthermore, for some of them the possibility constraint is relaxed and, by getting education, their utility increases.

### 4.5 Bequest taxation and average welfare

In this section, we investigate the relationship between bequest taxation and average economic variables, in particular average utility.

We proceed as follows: First of all, we consider that individuals, at each moment of time, can have skilled or unskilled mother and also herself be skilled or unskilled. Therefore, we have four different “kinds” of individuals. In a second step we will evaluate the average variables, as endowment, ability, effort for each group of individual. Finally, we will aggregate variables through the four groups, obtaining results for the whole economy.

The four possible kinds of individuals are : 1) skilled individuals with unskilled mothers, 2) skilled individuals with skilled mothers, 3) unskilled individuals with skilled mothers, 4) unskilled individuals with unskilled mothers.

Computing:

- the probability to be in each of the four states;
• the average endowment (\(\bar{\Sigma}\)) of skilled and unskilled individuals;
• the average ability (\(\bar{a}\)) of skilled and unskilled individuals;
• the average effort (\(\bar{x}\)) of skilled and unskilled individuals;

It turns out that the average talent of skilled individuals is increasing in bequest taxation (\(T\)) whereas the average talent of unskilled individuals is decreasing in \(T\). As expected, our allocation effect takes place: bequest taxation pushes more talented individuals toward the skilled positions.

We obtain the following result:

**Proposition 6 Welfare**

Redistributive policies based on bequest taxation can increase average utility. In particular, the average utility function exhibits a maximizing \(T^*\), with the following features:

If \(g = 0\), \(T^* = 1\) because bequest taxation “allocates” the more talented individuals in the skilled position without affecting the “psychological utility” coming to parents from leaving bequests.

If \(g = 1\), \(T^*\) can be both positive or negative because taxation heavily affects the “psychological utility” of individuals, and this effect may be stronger than the positive effect due to the better allocation of talents.

If \(0 < g < 1\), \(0 < T^* < 1\).

**Proof:** see appendix.

In our framework \(g\) is reasonably close to 0 (see paragraph 4.1). In that case, even considering the endogeneity of effort determination, we obtain that it exists a given level of bequest taxation which maximizes average utility.

This result is not surprising because it comes directly from what we called the allocation effect. Our economy is surely better off if the more talented individuals are those who get education because they spend less money in the educational process and because their contribution to the production process is higher.

### 4.6 Financing education

Different results emerge if we assume that bequest taxation, instead of being redistributed among all individuals as assumed above, is used to reduce education costs\(^{35}\) (our \(\xi\)), so that redistribution goes from people who leave bequest to people who get education. In that case, the cost of education becomes

---

\(^{35}\)Public education is usually financed by income taxation. This would reduce the cost of education, our \(\xi\), by taxing income produced by the same generation. We tried to analyze this
\((\xi - \frac{E(S_{t-1})}{S_{t-1}}) (1 - a_{i,t})\). Using the same utility function of equations 4 and 5, it is possible to show\(^3\):

**Proposition 7 Bequest taxation and education financing**

In the case that bequest taxation is used to finance education, the steady state equilibrium will remain unchanged because the possibility constraint is not affected by bequest taxation. Average utility may reduce with respect to \(T\) because of the “psychological effect”.

**Proof:** see appendix.

This surprising result may be intuitively explained as follow. In a first stage the cost of education is reduced, so that more people get education. The per-capita public spending on education reduces and the “possibility constraint” is newly strengthened. Given our definition of steady state (flows equilibrium, so constant number of skilled) anything changes because the possibility constraints of both skilled and unskilled are not affected by the tax rate. Therefore, redistributive policies (where fiscal income goes to all individuals) based on bequest taxation are more efficient in increasing the wellbeing of the economic system with respect to policies where bequest taxation is used in order to reduce the cost of education.

Checchi, Ichino and Rustichini (1999) try to solve the following puzzle: “why the Italian school system, which is strongly egalitarian in the quality and cost of education provided to rich and poor families, fails to generate at least the same degree of intergenerational mobility which prevails in the US, where the school system is instead highly decentralised and non egalitarian?” Their theoretical model suggests that a non standardized school system favours a better design of available education opportunities, favouring a better fit between the demand and supply of labor and, therefore, enhancing the returns of education, especially for children coming from poor families. While they present a solution involving “incentives”, our model refers to “constraints” and suggest that mobility and efficiency are favoured by intergenerational redistribution and not by a system “equal” in the sense that it assigns the same public expenditure to every individuals (as Checchi (2005) signals).

\(^3\)See appendix for proof.

\(^3\)In our model, but we have not been able to obtain analytical solution. From simulations we obtained the result that utility of individuals is always decreasing in income taxation.
5 Conclusions

A strong link between parents’ and children’s socio-economic status has been pointed out in many empirical studies on intergenerational mobility.

In this paper we focus on the linkages between the educational attainments of parents and the ones of children with the aim of checking the relationship between intergenerational mobility and allocational efficiency, which requires higher educational level to be attained by more talented individuals.

In section 3 we give some empirical evidence concerning intergenerational mobility and talents. Managing data from SHIW (Italy), BCS (UK) and NYLS (US), and using different proxies for talent, it turns out that children of unskilled individuals gain skilled positions with lower probability than children of skilled individuals.

Using different measure of talent (“scholastic” talent versus a “rough” proxy of “inborn” talent) we show that the ratio of misallocated individuals\(^{37}\) is between 2.3% and 12.7% in the different countries for children of unskilled individuals and between 3.2% and 17.9% for children of skilled individuals.

These results seem to confirm the Carneiro and Heckman (2002) findings of a 6% of individuals that, in the US, are credit rationed (short-run factors in their analysis).

The theoretical model presented in section 4 considers a world with heterogeneous agents endowed with different “inborn” talent. Each individual chooses her effort level, the amount of bequests and her educational attainment, which can be bounded by financial constraints, so that for some individuals schooling decisions are rationed. Children of skilled parents require a lower talent to get the highest educational level than children of unskilled parents; the allocation of talent is far from being efficient.

A proportional taxation on bequests \((T)\), whose yield is used for intergenerational redistribution, increases both the probability that a child of unskilled parents can achieve the skilled position and the probability that a child of a skilled mother becomes unskilled. Therefore, intergenerational educational mobility is increasing in bequest taxation.

Furthermore, even considering the endogeneity of effort determination and the negative impact of bequest taxation\(^{38}\) on “psychological utility”, the model indicates that bequest taxation raises average utility because of the allocational

\(^{37}\)Our misallocated individuals are: unskilled children of unskilled parents who would have been skilled, and skilled children of skilled parents who would have been unskilled, if the allocation in the skilled position was not dependent on parents condition.

\(^{38}\)Note 26 argues that model results overestimate the negative impact of taxation.
effect. Given that an economy is surely better off if the more talented individuals are those who get education, a programme of intergenerational redistribution via bequest taxation has a positive efficiency effect because it partly separates education from wealth.

We also show that this positive allocational effect does not arise if the fiscal yield coming from bequest taxation, instead of being distributed among all youngs, is devoted to finance education, so that it is distributed among students alone.

The model has shown that proportional bequest taxation increases both “equity” and “efficiency” of the economic system if its yields are used to redistribute among all individuals of the following generation, pushing the economic system toward an “equality of opportunities” world.
Appendix 1: Proofs

Proof 1 Definition of endogenous variables

Because of the logarithmic form of the \( f(\cdot) \) function, \( C_{i,t}^{JJ} = \rho S_{i,t}^{JJ} \) holds. Therefore, the first and second results are immediately obtainable from FOC’s.

The third result comes directly from the definition of \( \Sigma_{i,t}^{J} \) (see the first point after equation 1). In fact \( \frac{\Sigma_{i,t}^{J}}{\partial t} = E(S_{t-1}) - S_{t-1}^{JJ} \).

The last result comes from the comparison between equation 7 and 9. \( x_{i,t}^{JS} > x_{i,t}^{JU} \) if \( \xi(1 - a_{i,t}) > \Sigma_{i,t}^{J}(1 - \mu) \), where the term on the left is positive and the term on the right is negative.

Proof 2 The convenience condition

Using equations 7 and 9, the difference \( x_{i,t}^{JS} - x_{i,t}^{JU} \) in equation 12 depends on \( \Sigma_{i,t}^{J} \). Defining \( \phi_{t} \) the endogenous ratio of skilled individual in the population at time \( t \), the average bequest is \( E(S_{t-1}) = \phi_{t-1}S_{t-1}^{S} + (1 - \phi_{t-1})S_{t-1}^{U} \) and the endowment received by a generic child is:

\[
\Sigma_{i,t}^{J} = S_{i,t-1}(1 - T) + [\phi_{t-1}S_{t-1}^{S} + (1 - \phi_{t-1})S_{t-1}^{U}]T \quad \text{for} \quad J = S, U \quad (i)
\]

Substituting equations 7 and 9 in eq. 12, using eq. i, and solving for \( a_{i,t} \), we obtain \( U_{i,t}^{JS} > U_{i,t}^{JU} \) if, respectively for children of unskilled and skilled mothers:

\[
a_{i,t} > \hat{a}_{i,t}^{U} \equiv 1 - \frac{1 + \rho \mu \ln(\mu) - (\mu - 1)[\phi_{t-1}(\mu - 1)T + 1]}{m\mu} \quad (ii)
\]

\[
a_{i,t} > \hat{a}_{i,t}^{S} \equiv 1 - \frac{(\mu - 1)(1 - \phi_{t-1})(\mu - 1)T - \mu - 1 + \rho \mu \ln(\mu)}{m\mu} \quad (iii)
\]

Comparing the two thresholds, we immediately obtain \( \hat{a}_{i,t}^{U} < \hat{a}_{i,t}^{S} \) if \( (\mu - 1)^{2}(T - 1) < 0 \), which always holds because \( \mu > 1 \) and \( T < 1 \).

The last part of lemma 2 comes directly for the definition of \( \hat{a}_{i,t}^{S} \) and \( \hat{a}_{i,t}^{U} \).

Proof 3 The possibility condition

Children of unskilled mothers are not financially constrained in education if their talent is higher than a critical value which can be calculated solving in \( a_{i,t} \) the condition \( \Sigma_{i,t}^{U} \geq \xi(1 - a_{i,t}) \):

\[
a_{i,t} > \tilde{a}_{i,t}^{U} \equiv 1 - \frac{1 + (\mu - 1)\phi_{t-1}T}{m\mu} \quad (iv)
\]

\[39] In order to have \( 0 \leq \tilde{a}_{i,t}^{U} \leq 1 \), \( 0 \leq \tilde{a}_{i,t}^{S} \leq 1 \) some restrictions on parameter are needed. Nevertheless, as explained below, we will not use this thresholds so that we do not present these restrictions here.
whereas for children of skilled mothers the threshold becomes:

\[ a_{i,t} > \tilde{a}_{i} = 1 - \frac{\mu - (\mu - 1)(1 - \phi_{i-1})T}{m\mu} \] (v)

Where \(0 \leq \tilde{a}_{i} \leq 1\), \(0 \leq \tilde{a}_{U} \leq 1\) always holds. Furthermore, \(\tilde{a}_{i} \leq \tilde{a}_{U}\): if \((1 - T)(\mu - 1) \geq 0\), always verified. Differentiating equations iv and v with respect to \(T\), we obtain the result of the third part of lemma 3.

**Proof 4** The allocation effect

From equation 17 we have \(\frac{dp_{US}}{dT} = \frac{\mu - 1}{m\mu[m(\mu - 1) + 1]} \geq 0\);

From equation 18 we have \(\frac{dp_{SU}}{dT} = -\frac{(\mu - 1)(\mu - 1)}{m[m(\mu - 1) + 1]} \leq 0\).

**Proof 5** Individual utility

For an individual endowed with a given talent, the optimal effort depends on the amount received from the previous generation \(\Sigma_{i}^{J}\), defined in equation i, which in turn depends on the amount received both directly by the mother and by redistribution; considering \(S_{i-1} = \mu S_{i-1}\) (see equations 6 and 8) we obtain:

- for \(J = S\), so that for children of skilled mothers,

\[ \frac{d\Sigma_{i}^{S}}{dT} = -(\mu - 1)(1 - \phi)\frac{\rho}{\gamma} < 0 \] (vi)

- for \(J = U\), so that for children of unskilled mothers,

\[ \frac{d\Sigma_{i}^{U}}{dT} = (\mu - 1)\phi\frac{\rho}{\gamma} > 0 \] (vii)

First, we consider individuals with a skilled mother. Their utility depends on \(T\) in the following way:

\[ \frac{dU_{S}}{dT} = -\rho \frac{g}{1 - gT} - \gamma \frac{dx_{S}}{dT} \]

where \(x_{S}\) is defined in equations 7 and 9. Using equation vi, \(\frac{dU_{S}}{dT} < 0\) whichever is the skilled position.

Now, let us consider individuals with an unskilled mother. Their utility depends on \(T\) in the following way:

\[ \frac{dU_{U}}{dT} = -\rho \frac{g}{1 - gT} - \gamma \frac{dx_{U}}{dT} \]

If \(g = 0\), using equations 7 and 9, from equation vii) \(\frac{dU_{U}}{dT} > 0\) always hold.

Following the same steps, but with \(0 < g \leq 1\), \(\frac{dU_{U}}{dT}\) has an indeterminate sign because of the two opposite effects outlined in the previous equation. The utility
function exhibits a maximum in $T^{UJ}$, different between skilled and unskilled individuals because of the slightly dissimilar effect that $T$ produces on effort in the two cases (see 7 and 9). After some algebraic steps it emerges that $U^{UU}(T)$ presents a maximum for

$$T^{UU} = \frac{1}{g} - \frac{1}{\mu - 1} \frac{1}{\phi^*(m, \mu)}$$

and $U^{US}(T)$ presents a maximum for

$$T^{US} = \frac{1}{g} - \frac{\mu}{\mu - 1} \frac{1}{\phi^*(m, \mu)}$$

In the case of unskilled with unskilled mother, $T^{UU} > 0$ if $g < (\mu - 1)\phi(m, \mu)$; in the case of skilled with unskilled mother, $T^{US} > 0$ if $g < (\frac{\mu - 1}{\mu})\phi(m, \mu)$.

**Proof 6 Welfare**

From equations 10 and 11, average utility in the whole population is:

$$\overline{U} = \phi(1 + \rho)\ln \left[\frac{\mu}{\gamma}\right] + \rho \ln(1 - gT) - \gamma((1 - \phi)x^U + \phi x^S)$$  (viii)

where $x^J$ for $J = U, S$, is the average effort of unskilled and skilled individuals.

In steady state equilibrium, from equation 9 and 7, the average effort of skilled and unskilled workers is given by:

$$x^S = 1 + \frac{\rho \gamma}{\mu} - \frac{\rho + \gamma}{\mu} \Sigma^S + \left(\xi + \mu \lambda\right)\pi^S$$  (ix)

$$x^U = 1 + \frac{\rho \gamma}{\mu} - \left(\Sigma^U + \lambda \pi^U\right)$$  (x)

where $\Sigma^J$ and $\pi^J$, for $J = U, S$ are respectively the average values of endowment received from the previous generation and the average talent of individuals.

In order to calculate $\frac{d\overline{U}}{dT}$, we need to calculate $\frac{d\pi^J}{dT}$, for $J = U, S$. From equations ix and x, these derivatives depends on derivatives of $\Sigma^J$ and $\pi^J$ with respect to $T$. The following part is devoted to calculate these derivatives.

For skilled individuals, the probability of having an unskilled mother is:

$$q^{US} = \frac{(1 - \phi^*(m, \mu))p^{US}}{\phi^*(m, \mu)p^{SS} + (1 - \phi^*(m, \mu))p^{US}} = p^{SU}$$

where the last term is obtained substituting the definition of $\phi$ of equation 1440.

Furthermore, in steady state the number of stayers must be constant, so that $q^{SS} = p^{SS}$ and $q^{UU} = p^{UU}$.

In fact, $q^{US}$ is the probability of having $U$ mother conditional to be a $S$ individual. $p^{SU}$ is the probability of being an individual of type $U$, conditional to having a mother of type $S$, hence the two probabilities refer to the same stock of individuals. In steady state the number of movers between the two states must be the same, so that $q^{US} = p^{SU}$, $q^{SU} = p^{US}$, and $q^{UU} = p^{UU}$.
A skilled individual will receive the endowment of equation i, for \( J = S \), with probability \( q_{SS} \) and the same endowment but for \( J = U \) with probability \( q_{US} \).

To keep notation simple, let us define

\[
\Theta(T) = \left( \frac{(\mu-1)(1-T)}{m\mu} \right)^2
\]

where \( \frac{d\Theta}{dT} < 0 \).

With some algebraic steps we obtain the average endowment of skilled individuals:

\[
\Sigma^S = [1 + \mu(m-1)\Theta(T)] \phi^*(m, \mu) \xi
\]

with

\[
\frac{d\Sigma^S}{dT} = \phi^*(m, \mu) \xi \mu(m-1) \frac{d\Theta}{dT} \leq 0
\]

An unskilled individual will receive the endowment of equation i, for \( J = U \) with probability \( q_{UU} \) and the same endowment but for \( J = S \) with probability \( q_{SU} \), so that the average endowment of an unskilled individual is:

\[
\Sigma^U = [1 - \Theta(T)] \phi^*(m, \mu) \xi
\]

with

\[
\frac{d\Sigma^U}{dT} = -\phi \xi \frac{d\Theta}{dT} \geq 0
\]

Given \( \bar{a}^S \) and \( \bar{a}^U \) (equations v and iv), and given the hypothesis of uniform distribution of talent with support on \([0,1]\), we can easily compute the average talent of children of unskilled mothers who remain unskilled (\( \bar{a}^{SU} \)) and the average talent of the ones who become skilled (\( \bar{a}^{SU} \)); the same for children of skilled mothers (respectively, \( \frac{\bar{a}^U}{2} \) if they become unskilled, and \( \frac{1+\bar{a}^U}{2} \) if they get the skilled position.). The \( q^{Jj} \) probabilities, for \( J = U, S \), allows us to compute average talent of the skilled (\( \bar{a}^S \)) and of the unskilled (\( \bar{a}^U \)):

\[
\bar{a}^S = \frac{\phi^*(m, \mu)}{2} \left[ \mu(m-1)(2 - \Theta(T)) + 1 \right]
\]

\[
\bar{a}^U = \frac{\phi^*(m, \mu)}{2} \left[ \mu(m-1) + \Theta(T) \right]
\]

The average talent of skilled individuals is increasing in bequest taxation (\( T \)) whereas the average talent of unskilled individuals is decreasing in \( T \).

In fact,

\[
\frac{d\bar{a}^S}{dT} = \frac{\phi^*(m, \mu)}{2} \left[ \mu(m-1) \left( -\frac{d\Theta}{dT} \right) \right]
\]
and
\[
\frac{\partial U}{\partial T} = \frac{\phi^*(m, \mu)}{2} \left( \frac{d\Theta}{dT} \right)
\]  
(xviii)

From equations x, ix, xii, xiv, xvii and xviii:

\[
\frac{\partial x^S}{\partial T} = -\phi^*(m, \mu)(m-1)\frac{d\Theta \xi - \mu \lambda}{2}
\]

\[
\frac{\partial x^U}{\partial T} = \left[ \xi - \frac{\lambda}{2} \right] \frac{d\Theta}{dT} \phi^*(m, \mu)
\]

Plugging these last derivatives into equation viii we finally obtain:

\[
\frac{\partial U}{\partial T} = \rho \left[ \phi^*(m, \mu)^2 (\mu - 1)^2 (2\mu - 1) \frac{m-1}{m} (1-T) - \frac{g}{1-gT} \right]
\]

Defining:

\[
M(m, \mu) = \phi^*(m, \mu)^2 (\mu - 1)^2 (2\mu - 1) \frac{m-1}{m}
\]

we obtain

\[
\frac{\partial U}{\partial T} = \rho \left[ (m, \mu)[M(1-T) - \frac{g}{1-gT}] \right]
\]

For \( g = 0 \) it turns out that the average utility exhibits a maximum \( T = 1 \).

If \( g > 0 \) the utility is maximized for \( \star \):

\[
T^* = \frac{1}{2} \left[ 1 + g \frac{g}{g} - \sqrt{\left( 1 - \frac{g}{g} \right)^2 + \frac{4}{M(m, \mu)}} \right]
\]

In order to have \( T^* < 1 \) we need that \( g < M \). This condition assures that an optimal bequest taxation exists.

**Proof 7** Bequest taxation and education financing

Assume that fiscal yield given by \( \frac{E(S_{t-1})}{\phi_t-1} \) is used to finance education, so that consumption of skilled individual is given by:

\[
C_{i,t}^J = S_{i,t-1} - \left( \xi - \frac{E(S_{t-1})}{\phi_t-1} \right) (1-a_{i,t}) + \mu(x_{i,t}^{JS} + \lambda a_{i,t}) - S_{i,t}^{JS} \quad \text{for } J = U, S \quad (xix)
\]

whereas consumption for the unskilled is:

\[
C_{i,t}^J = S_{i,t-1} + (x_{i,t}^{JU} + \lambda a_{i,t}) - S_{i,t}^{JU} \quad \text{for } J = U, S \quad (xx)
\]

and that the utility function is the one presented in equation 3. From FOCs we can define the optimal level for choice variables and the indirect utility both for

\footnote{Given \( \frac{1+g}{g} \geq 2 \) for \( 0 \leq g \leq 1 \), we can consider the root with minus sign alone.}
skilled and unskilled individuals. We obtain that $S_{i,t}^J$, for $J = S,U$, is equal to the one defined in equation 6 and 8 whereas the different type of redistribution modifies the optimal effort.

In the hypothesis that the possibility constraint holds, we can write it for both skilled and unskilled individuals:

$$S_{t-1}^J(1 - T) - \left( \xi - \frac{E(S_{t-1})}{\phi_{t-1}} \right) (1 - a_{i,t}) > 0 \quad \text{for} \quad J = S,U$$

where $E(S_{t-1}) = \phi_{t-1} \frac{\mu}{\gamma} + (1 - \phi_{t-1}) \frac{\xi}{\gamma}$. The minimum level of ability which allows individuals to obtain education becomes:

$$\tilde{a}^S = 1 - \frac{\mu(1 - T)}{m\mu - \left( \frac{1}{\phi_{t-1}} + \mu - 1 \right) T} \quad (\text{xxi})$$

$$\tilde{a}^U = 1 - \frac{1 - T}{m\mu - \left( \frac{1}{\phi_{t-1}} + \mu - 1 \right) T} \quad (\text{xxii})$$

Given these two thresholds, the transition probabilities are the ones defined in equation 15 and the definition of $\phi$ is the same as the one in equation 14. Therefore, we can solve for the steady state quota of skilled $\phi^*(m, \mu)$, obtaining that the steady state quota of skilled is the same of equation 16, and substitute it into equations xxi and xxii, obtaining:

$$\tilde{a}^S = 1 - \frac{1}{m}$$

and

$$\tilde{a}^U = 1 - \frac{1}{m\mu}$$

which are independent on the tax rate $T$. 

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