EVALUATING THE INCOME TAX REDISTRIBUTIVE EFFECTS,
A MORE COMPREHENSIVE APPROACH

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Abstract

In order to investigate redistributional effects induced by personal income tax (PIT), empirical researchers are
used to fixing a common distribution of income before tax for all tax schedules being compared. On one hand,
this approach allows to isolate the effect of tax policies. On the other hand, they do not deal with the fact that
the income tax redistributive effect is mainly determined by the matching between the pre-tax income
distribution and the tax schedule. This paper presents a simple procedure useful for practitioners wishing to
take into account pre-tax distributional differences before undertaking comparisons between tax schedules, in
particular with regard to the analysis of tax reforms which will act on unknown and/or coming pre-tax income
distributions. This new approach is illustrated for the Italian PIT case, according to the comparison of the
2005 tax reform versus the 2000 tax law.

JEL Codes: H23, H24, D63
Key words: personal income taxation, redistribution, welfare

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1. INTRODUCTION

In order to investigate redistributional effects induced by personal income tax (PIT) systems, and in accordance with the standard result on redistribution of Jakobsson (1976) and Kakwani (1977a), empirical researchers are used to fixing a common distribution of income before tax for all tax schedules being compared. Even if the local ordering of schedules is equivalent to the Lorenz quasi-ordering, it does not take into account the before tax inequality and the possibility to be dependent, with reference to outcomes, on the before tax distribution chosen as the “reference” distribution.

On one hand, this approach allows to isolate the effect of tax policies. On the other hand, it does not deal with the fact that the income tax redistributive effect is mainly determined by the matching between the pre-tax income distribution and the tax schedule operating on this distribution. Is this realistic for accurate intertemporal - and/or international - comparisons when, as a matter of fact, what matters for the distributional impact of income taxation is the tax structure and where the taxpayers are located? In accordance with the fact that there is definitely a lot of evidence relating to pre-tax income distributive changes over time, and across nations, if the goal is also to assess the actual impact rather than the potential effects of various tax regimes, an appropriate redistribution analysis should require incorporation of pre-tax inequality differences.

Dardanoni and Lambert (2002; shortly, DL) were able to formulate what could be a strategy a practitioner needs to perform if she or he wishes to take pre-tax inequality into account drawing out correct and real distributional implications of tax reforms. To summarise, their procedure acts on the pre- and post-tax distributions under analysis: it looks for an isoelastic transformation between the pre-tax distributions, i.e. before and after reform, and, if this were the case, corrects for the effect of the pre-tax distributional differences between the post-tax distributions, i.e. before and after reform.1

It follows that standard result on redistribution of Jakobsson and Kakwani are preserved under specific conditions on the structure of pre-tax income distributions and it achieves an “independence of baseline” property. Notice that if micro-data regarding the distributions of pre-tax incomes are known and available for the practitioners, the DL’s procedure may be certainly appropriate for intertemporal - or intercountries - comparisons.

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1 A transformation $g$ is isoelastic if $\exists A, b > 0 : g(x) = A x^b$. 
In this paper, I focus on an interesting insight. If an isoelastic transformation does hold between pre-tax income distributions through time (e.g., in a nation), it seems reasonable enough to conclude that this will occur again. As a consequence, with some parameters reflecting size and scale of inequality differences, say $A$ and $b$, respectively, there should exist some isoelastic transformation able to transplant an unknown and coming pre-tax income distribution into an already known one. For instance, and excluding the impact of exogenous structural crisis factors (however, these could affect all the population with, roughly, the same proportion), in accordance with this hypothesis an empirical researcher wishing to assess, now, which may be the outcomes of a tax reform, could proceed in two stages. Firstly, as usual, she or he can simulate those outcomes by using the most recent available distribution of pre-tax income. Secondly, in addition, she or he may take into account distributional differences, for instance by assuming different values for the parameters $A$ and $b$. By this way, we may control for a range of potential distributional variations: the choice of parameter values is the factor which could influence, together with the new tax structure, the potential redistributive effect.

In this paper, I exclusively focus on the second stage. In particular, I propose a methodology where comparisons of redistributive effects are based on the (relative) Lorenz criterion (thus, the Atkinson’s theorem is helpful to derive normative significance in a pure income distribution model), and a typical effectual progression measure, the Reynolds-Smolenky redistributive effect index (see Reynolds-Smolenky, 1977). Both tools are of course well-known, nevertheless, as far as I know, following this method tax systems intertemporal comparisons have not been treated elsewhere before.

The rest of the paper is as follows. Section 2 presents the analytical framework and most popular tools which are on hand to empirical researchers. In section 3 I present the methodology, discussing the issues and choices involved in implementing this innovative procedure. I propose the 2005 Italian PIT (IRPEF) reform as a good candidate for this more comprehensive approach, and, in this section, also the original data set and some general methodological issues about the microsimulation model that provides the data are presented.

In the concluding section, I present and evaluate the main pure redistributive results with regard to the 2005 IRPEF (Tremonti’s reform) versus the 2000 IRPEF (Visco’s reform).\(^2\)

\(^2\) The IRPEF reform of 2005 seems to be a good choice, several practitioners have evaluated redistributive effects by using the standard methodology (among others, cf. Arachi-Zanardi, 2002; Baldini-Bosi, 2000, 2002, 2004a, 2004b; Baldini et al., 2002, 2006; Schioppa, 2002; Gastaldi-Liberati, 2004; Declich et al., 2005; Tondani-
According to these, there is room to partially discuss the “potential” redistributive gains of the Italian PIT reform. Finally, I discuss in the light of my results the well-known Musgrave and Thin (1948, p. 510) expectation, “ […] the less equal the distribution of income before tax, the more potent will be a (given) progressive tax in equalizing income”, and offer some final remarks.

2. SOME DEFINITIONS AND TOOLS

First, I present the tools which have been the usual reference in the practical work of assessing the redistributive impact of alternative tax systems.

Thus, let original income \( x \) be continually distributed over some support \([0, z]\) and represented by the function \( F : [0, z] \rightarrow [0, 1] \); the pre-tax income distribution function is denoted by \( F(x) \) and \( f(x) \) is the associated density function defined on the same interval and assumed strictly positive throughout the support from the lowest income \( x_{\text{min}} \geq 0 \) to the highest income \( x_{\text{max}} \leq z \) (\( z \) is any income level in excess of the highest one that actually occurred); \( n \) is the number of the income-receiving units. For each \( p \in [0, 1] \) there is just one income level \( y \), which satisfies \( p = F(y) \). This means that the first 100\( p \)-percent of income units are those with pre-tax income less than or equal to \( y \).

The \( T(x) \) is the tax function of an income unit having pre-tax monetary income \( x \) and will be assumed twice differentiable. I denote \( T'(x) \) as the first derivative and assume that \( 0 \leq T'(x) < 1, \forall x \), thus \( 0 \leq T(x) < x, \forall x \).\(^3\) Notice that the individual tax burden is a function only of the monetary income while a typical income tax structure is also a function of other features (I return to this later in the paper). The \( T(x) \) function characterisation implies that the disposable income \( N(x) = x - T(x) \) is a monotonic non-decreasing function of pre-tax income \( x \). The mean pre-tax income, mean tax liability and mean post-tax income are, respectively,

\[
\mu_X = \int_0^z x f(x) \, dx, \quad \mu_T = \int_0^z T(x) f(x) \, dx, \quad \text{and} \quad \mu_N = \int_0^z N(x) f(x) \, dx,
\]

and the total tax ratio is \( \frac{\mu_T}{\mu_X} = t \).

\(^3\) An increasing with income average tax rate \( t(x)/x \) is assumed to be the condition for (weak) progression:

\[
d \left[ \frac{T(x)}{x} \right] / dx \geq 0 \ \forall x \iff T'(x) \geq T(x) / x \ \forall x > 0.
\]
The Lorenz curve is widely regarded as the most general of all inequality measures. To present the Lorenz order consider the Lorenz function \( L : [0, 1] \to [0, 1] \); \( L_X \), \( L_N \), and \( L_T \) will refer respectively to the Lorenz function for the pre-tax income, post–tax income and tax liability,

\[
p = \rightarrow F(y) \quad L_X (p) = \left( \frac{1}{\mu_X} \right) \int_0^y x f(x) \mathrm{d}x,
\]

\[
p = \rightarrow F(y) \quad L_N (p) = \left\{ \frac{1}{\mu_X (1 - t)} \right\} \int_0^y N(x) f(x) \mathrm{d}x,
\]

\[
p = \rightarrow F(y) \quad L_T (p) = \left[ \frac{1}{\mu_X t} \right] \int_0^y T(x) f(x) \mathrm{d}x.
\]

The graph of a Lorenz function is the (conventional) Lorenz curve, which indicates the share of total income enjoyed by the bottom \( p \) proportion of the population, ordered by income from lowest to highest.\(^4\) For the sake of income distribution comparisons, the Lorenz curve always closer to the 45° line is said to represent less inequality.

In two seminal papers in the static literature, Jakobsson (1976) and Fellman (1976) point out that:

\[
\frac{d}{dx} \left[ T(x)/x \right] \geq 0 \quad \forall x \quad \Leftrightarrow \quad L_N (p) \geq L_X (p) \geq L_T (p) \quad \forall F(x).
\]

As a consequence, with a tax code designed for any homogenous sub-population where the only difference among people are the income levels and \( t > 0 \), a progressive income tax is within-group-inequality reducing according to the dominance of post-tax income Lorenz curve over the pre-tax income Lorenz curve (the latter should be nowhere above the former and at least somewhere strictly below).

I highlight the fact that according to different pre-tax income distributions, the application of a progressive income tax could lead to different outcomes: we want to know, for instance, if the result of the comparison of two or more tax schedules could be counteracted by the shift in the distribution of pre-tax income, \textit{i.e.}, by changing the reference pre-tax distribution.

Thus, the literature has coherently offered two main ways to measure the redistributive power of tax systems. The \textit{measure of effective progression} (or \textit{progressivity}) that is employed in the current analysis is the Reynolds-Smolensky (\( \Pi^{RS} \)) index:\(^5\)

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\(^4\) The \( T(x) \) function characterisation allows us to consider the \( L_N \) and \( L_T \) concentration curves as Lorenz curves.

\(^5\) \textit{Cf.}, among others, Blackorby-Donaldson (1984), Kakwani (1977a) and Suits (1977) with regard to other global measures of progressivity.
\[ \Pi^{RS} = 2 \int_0^1 [L_N(p) - L_X(p)] \, dp = G_X - C_N \quad (\geq 0 \text{ with progressive income tax}), \]

where \( C_N = \text{post-tax income concentration coefficient} = 1 - 2 \int_0^1 L_N(p) \, dp, \]

and \( G_X = \text{pre-tax income Gini coefficient} = 1 - 2 \int_0^1 L_X(p) \, dp. \)

According to a redistributional effect procedure, the higher the Reynolds–Smolensky index the more equal could be considered the post-tax income distribution with regard the distribution defined by the pre-tax Lorenz curve (notice that the latter can be interpreted as the distribution of after-tax incomes resulting from an - equal yield - flat tax). Given the assumptions of this section, \( \Pi^{RS} \) measures the Gini coefficient reduction following the taxation process.

Due to reason of transparency with respect to other tools below outlined, I also present one of the most used local measures of structural progression.\(^7\)

Let denote residual progression at income \( x, \RP(x) \), as the elasticity of the income after tax with respect to income before tax; a necessary and sufficient condition for the existence of non-negative redistribution is \( 0 \leq \RP(x) \leq 1, \forall x. \)

According to Jakobsson (1976) and Kakwani (1977a)\(^8\) (shortly, JK), given any particular distribution of pre-tax income, say \( F(x) \), let \( N_1(x) \) and \( N_2(x) \) be two post-tax income schedules induced by their respective tax liabilities, \( T_1(x) \) and \( T_2(x) \):

\[ \RP_1(x) \leq \RP_2(x) \quad \forall x \quad \iff \quad L_N^1(p) \geq L_N^2(p) \quad \forall F(x). \]

Thus, according to this local measure, lower residual progression implies, and is implied by, higher progressivity. Whenever the pre-tax distribution remains the same for all schedules being compared, the local ordering of schedules is equivalent to the Lorenz partial ordering (a local-to-global comparison).

Within this static framework, this is the key point which DL’s procedure started from.

If practitioners wished to take pre-tax inequality into account and to assess the real impact rather than potential effects of various systems, two stages are required for DL’s procedure. Firstly, as usual, conclusions should not be sensitivity to the choice of the reference distribution. DL show that “the (residual) progression comparisons can be guaranteed

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\(^6\) Notice that \( C_N = G_N \), see footnote 4, \textit{ivi}.

\(^7\) On this, \textit{cf.} the seminal article of Musgrave and Thin (1948).

\(^8\) On this, \textit{cf.} also Hemming-Keen (1983) and Latham (1988).
invariant to the choice of baseline if and only if the candidate reference distributions are isoelastic transformations of one another” (DL, 2002, p. 105). Secondly, they show that a transplant-and-compare procedure is relevant to draw out correct distributional implications.

Following DL (ib.), let \( g : \mathbb{R}_+ \to \mathbb{R}_+ \) be any monotonic increasing function.

In terms of \( g \), let us define the deformation \( N^g \) of a post-tax income schedule \( N \),

\[
N^g = g \circ N \circ g^{-1},
\]

and the deformation \( \langle N,F \rangle^g \) of a generic regime \( \langle N,F \rangle \) consisting of an actual post-tax schedule and pre-tax income distribution pair,

\[
\langle N,F \rangle^g = \langle N^g,F \circ g^{-1} \rangle.
\]

Notice that the function \( g \) operates a variable shrink (or stretch) of pre-tax relative income differentials. It follows obviously, with two regimes \( \langle N_1,F_1 \rangle, \langle N_2,F_2 \rangle \), and a “reference” distribution, call it \( F_0 \), that:

\[
g_i = F_0^{-1} \circ F_i \quad \Rightarrow \quad \langle N_i,F_i \rangle^{g_i} = \langle N_i^{g_i},F_0 \rangle \quad \text{for } i = 1, 2.
\]

To transplant two pre-tax income distributions into a common base distribution, some (probably different) appropriate transformation functions \( g_i \)’s do exist. As we shall see below for analytical details, the respective transformation functions \( g_i \)’s themselves should be used to correct post-tax relative income distributions and, by the comparisons between the transplanted regimes \( \langle N_i^{g_i},F_0 \rangle \)’s, it could be possible to achieve unambiguous local progression comparison between \( N_1^{g_i} \) and \( N_2^{g_i} \), if any, that can be represented as a partial progressivity ordering over regimes conditioned by \( F_0 \).

The authors show that the isoelasticity condition regarding to any possible reference distribution is a crucial point. In fact, a natural question to ask is whether the same result obtained by using \( F_0 \) may be found selecting another baseline, say \( G_0 \).

Let \( F_0 \) and \( G_0 \) be two alternative reference distributions for the comparison of regimes, \( \langle N_1,F_1 \rangle \) and \( \langle N_2,F_2 \rangle \). DL state (Theorem 1, p. 105) that:
the partial orderings over \( \langle N_1, F_1 \rangle \) and \( \langle N_2, F_2 \rangle \) conditioned on \( F_0 \) and \( G_0 \) are the same \( \iff \) 
\( G_0^{-1} \circ F_0 = g \) is isoelastic \( \iff \exists A, b > 0 : g(x) = A x^b \).

If the analyst were interested to transplant one distribution, \( F_1 \), directly into another, say \( F_2 \), to avoid the risk to be dependent on the elected baseline about findings, as a consequence she or he should verify if they are isoelastically linked.

Theorem 2 (DL, 2002, pp. 105-106) formally states what is on hand to practitioners. Let \( \langle N_1, F_1 \rangle \) and \( \langle N_2, F_2 \rangle \) be two regimes. The partial orderings over regimes conditioned by a generic reference distribution \( F \) is denoted by \( \phi_{P|F} \):

a) Let be \( F_0 \) any income distribution such that \( g_1 = F_0^{-1} \circ F_1 \) and \( g_2 = F_0^{-1} \circ F_2 \) are both isoelastic.

If \( \forall x \in \mathbb{R}^+ \quad \text{RP}(g_1(x)) \leq \text{RP}(g_2(x)) \) then \( \langle N_1, F_1 \rangle \phi_{P|F_0} \langle N_2, F_2 \rangle \); 

b) Assume that \( g = F_1^{-1} \circ F_2 \) is isoelastic.

If \( \forall x \in \mathbb{R}^+ \quad \text{RP}(g(x)) \leq \text{RP}(x) \) then \( \langle N_1, F_1 \rangle \phi_{P|F_1} \langle N_2, F_2 \rangle \phi_{P|F_2} \langle N_2, F_2 \rangle \); 

c) If \( g = F_1^{-1} \circ F_2 \) is not isoelastic, the partial orderings over regimes by \( \phi_{P|F_1} \) and \( \phi_{P|F_2} \) are different.

Parts a) and b) of this theorem lead to give relevance to the isoelasticity conditions issue, and in this way the potential for dependency of end results on the baseline is avoided. If this were not the case, the part c) affirms that conclusions may be questionable, reflecting the distribution, \( F_1 \) or \( F_2 \), selected as baseline. The practitioner should verify by making successive pairwise tests by using all the potential reference distributions under analysis whether outcomes are distribution-dependent, or not. Empirical researchers should be interested in parts a) and b) of this theorem: when one of them is verified it is straightforward to make use of JK results to infer the occurrence of Lorenz curves intersections.

On the other hand, looking directly at the relative Lorenz criterion it is straightforward to take into account pre-tax inequality. Let denote the Lorenz partial ordering of regimes by \( \phi_L \). The Lorenz dominance criterion states that:

\[ \langle N_1, F_1 \rangle \phi_L \langle N_2, F_2 \rangle \iff L_N^*(p) - L_N^*(p) \geq L_N^*(p) - L_N^*(p) \quad \forall p, \text{ with } > \text{ for some } p. \]
In accordance with this notation, it is possible to write the JK results in a slightly different way. Let denote by $\phi_{RP}$ the RP partial ordering of disposable income $N$:

$$N_1 \phi_{RP} N_2 \iff \langle N_1, F_0 \rangle \phi L \langle N_2, F_0 \rangle \quad \forall F_0.$$

If net income schedule yields are the same for all comparisons and Lorenz curves do not cross, the Atkinson’s theorem is helpful to derive normative significance: the Lorenz-dominating distribution is welfare superior.\(^9\)

In order to obtain a ranking of income distributions with respect to income inequality, Atkinson assumes an additively separable and symmetric Social Welfare Function:\(^{10}\)

$$W = \frac{1}{y} \int U(y) f(y) dy,$$

where $U$ is an evaluation function of post-tax incomes, $y$. Let $H(y)$ and $G(y)$ be two post-tax income distributions with equal means, $\mu_H = \mu_G$. Then,

$$L_H(p) \geq L_G(p) \quad \forall p \iff W_H \geq W_G \quad \forall U(y),$$

where $U'(y) > 0$, $U''(y) < 0$, $\forall y > 0$.

If the practitioner is ready to assume a Social Welfare Function coherent with the Pareto-criteria and inequality-adverse, inequality is simply a welfare loss.\(^{11}\)

Endowed with these tools, I turn to illustrate the main contribution of this paper.

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\(^9\) Non-equal yield taxes are usually the outcome of a personal income tax reform. According to an appropriate residual progression neutral tax device, I standardise the different total tax burdens (Pfähler 1984; Lambert-Pfähler, 1987). With a RP-neutral tax cut/hike, the gain/loss is the same for every sample observation in percentage terms; for every $p$, $RP(x)$ remains constant; the Lorenz curves, with respect to the post-tax income distributions under analysis - before and after the RP-neutral tax cut/hike - are exactly superimposed. The size of the cake changes, not how the shares are divided. See Chakravarty and Muliere (2003), about correct procedures to rank inequality.

\(^{10}\) Notice that I present both the Lorenz dominance analysis and a measure of effective progression, but I prefer a quasi-ordering among the set of income distributions by unanimous preference, rather than a complete ordering. Advocating the fundamental Atkinson’s theorem (1970), Formby and Smith (1986, p. 562) comment, “If Lorenz curves intersect, a social welfare function can always be found which ranks income distribution differently than does the Gini coefficient or other summary measures of inequality.”. If this were not the case, any inequality index that fulfils the Pigou-Dalton transfer principle and Symmetry will be robust (see, e.g., Foster, 1985). See Kondor (1975) about “value judgements implied by the use of various measures of income inequality”.

\(^{11}\) Notice that an (equal yield) proportional tax system is redistributionally neutral in the Lorenz sense:

$$p = F(y) \rightarrow L_N(p) = \left\{ \frac{1}{\mu_X (1 - t)} \right\} \int (x - tx) f(x) dx = L_X(p) = \left( \frac{1}{\mu_X} \right) \int x f(x) dx.$$

then, even if positive taxation per se - proportional or progressive - is only social welfare reducing, nevertheless, a progressive income tax is social welfare reducing by less than a proportional tax raising the same revenue from the same before tax income distribution.
3. PRE-TAX DISTRIBUTIONAL DIFFERENCES AND PROGRESSIVITY: A SIMPLE NEW METHODOLOGY

Critically, in Russo (2005) I proposed an application to Italian micro-data and PIT systems between 1995 and 2000 and, according to Dardanoni and Lambert’s requirements, I found that before tax log distributions differ essentially only by location and scale. By the occurrence of (various) isoelastic transformations, and the application of the transplant-and-compare procedure, followed a definitive result with respect to the actual redistributive impact of the 1998 and 2000 IRPEF reforms.

On the other hand, some failings of this procedure came out. As I already advised, actual redistributive effects and progressivity of income taxation strictly depend on the tax schedule and where the taxpayers are located. This matter is obviously cogent for practitioners involved in the evaluation of tax structures operating on known distributions of pre-tax income, i.e., where the micro-data produced by the microsimulation model are in her/his hand for all the income distributions under consideration. Also a realistic assessment about the different magnitude of the redistributive power that characterises, for instance, the current tax law and the proposal of reform needs to take into account pre-tax disparity: due to the missing of the required information, the DL’s procedure does not seem to be helpful and, as a consequence, the answer to this simple requirement is more demanding.

On the other hand, it is possible to develop a distinct and complementary redistributive effects analysis.

Practitioners paying attention to the possible repercussions of an unknown distribution of pre-tax income on the post-tax income distribution after shaping from a new tax scheme should firstly looks for the occurrence of isoelastic transformations between known pre-tax distributions: if this were the case, it seems reasonable enough to conclude that this will occur again (notice that the higher the number of disposable micro-data over time, the better is the degree of confidence for this assertion).

Under a specific condition with regard to the structure of pre-tax income distributions – of course, distinct estimated $A$ and $b$, but a substantially common structural link through time between pre-tax distributions – it is possible to argue that with, probably, different isoelastic parameters this relationship will continue still to exist, at least for a time sufficiently close to the most recent sample of the pre-tax distribution.
For a deeper evaluation of tax systems suffering by the missed knowledge with respect to the pre-tax distribution which the tax law will act on, a more comprehensive methodology is crucial. I propose that such an analysis should be characterised by two stages:

i) firstly, by using the most recent available distribution of pre-tax income and assuming that the underlying distribution has not changed over time, a practitioner can, as usual, simulate the outcome of a tax reform;

ii) Secondly, in addition, she or he may make use of a complementary methodology; always searching for potential redistributive effects, she or he can take into account distributional differences by assuming different values for the parameters \( A \) and \( b \) acting on the pre-tax distribution at hand.

By the assumption of the occurrence of an isoelastic transformation between the pre-tax distributions, in such a way it may control for a range of potential distributional variations: the choice of parameter values, measuring the change in size, \( A \), and scale of inequality, \( b \), is the factor which could influence, together with the new tax structure, the redistributive effect.

If the length of time series were large enough, a good approximation could be a simple average of preceding estimates (moreover, nominal growth rates data - or their forecasts - are widely available). Otherwise, if this length were not judged sufficiently large, by using different parameter values (sensitivity analysis) - arbitrary but economically reasonable - the practitioner could still verify the robustness of the evaluation based on the stage i).

In this paper, I exclusively focus on the stage ii) and in the next subsection it will be fully developed. In accordance with the sensitivity analysis, this is made by directly proposing an application of this methodology to PIT changes between 2000 and 2005.

For the assessment of redistributive effect of the 2005 IRPEF reform versus the tax scheme of 2000, the data set is based on the Survey of Households’ Income and Wealth (shortly, SHIW) published by the Bank of Italy. The SHIW, as most large-scale national surveys, allows a good deal of disaggregation and is widely used in empirical analyses of income and wealth in Italy. Thus, micro level information on different sources of disposable incomes, consumption, saving, monetary and financial variables, labour market, social-demographic characteristics of each household member, etc, are derived from the 2000-survey: it covers 8,001 households composed of 22,268 individuals.\(^{12}\) Households are randomly selected. The

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\(^{12}\) For a critical discussion about the SHIW data source see Brandolini (1999). Brandolini and Cannari (1994) analysed the quality of SHIW and advocated that it is similar to analogous surveys in other countries.
sampling design involves unequal stratum sampling fractions, thus, I need to use sampling weights to obtain unbiased estimates: by the SHIW data, to each household has been attached a sample weight inversely related to the probability to be included into the sample.\textsuperscript{13} I need to work with pre- and post-tax personal income distribution, so I need to recover the pre-tax incomes, since all data in the Survey are net of taxes.

I take advantage of micro-data produced by the “Istituto per gli Studi e l’Analisi Economica” (ISAE) with ITAXMOD package. ITAXMOD is a static microsimulation model that allows the simulation of the immediate impact of a change in tax policy rules. A description of the model is provided in Di Biase et al. (1995). It was, in 1989, the first microsimulation model of personal income taxation in Italy.\textsuperscript{14} In accordance with a new SHIW of the Bank of Italy, ITAXMOD acquires the post-tax income data revealed by the interviewed and embodies a procedure to reconstruct gross income, correcting for tax evasion under the assumption that the surveyed net income is halfway between the (minimum) after-tax declared income and the “true” net income.\textsuperscript{15} ITAXMOD developers postulate that the tax evasion is substantially concentrated on self-employment income, while wage and salary earners declared incomes assumed to be near the “true” values, thus, with an evasion parameter equal to zero. Then, after the application of the procedure, essentially based on the inverse of the algorithm that determines the individual tax burden, ITAXMOD provides gross income micro-data that are validated by exogenous information on fiscal (the Finance Ministry’s fiscal data stored by SOGEI) and national aggregates. Pre- and post-tax incomes so computed are thus the starting line for the application, in accordance with empirical analyses purposes, of various methodologies.\textsuperscript{16}

Hence, according to the ITAXMOD00 tax code, I have on hand both 2000 pre- and post-tax income distributions.

The 2000 pre-tax incomes so computed are the starting point to simulate the 2005 pre-tax distribution of income. According to the sensitivity analysis, in order to simulate different

\textsuperscript{13} As there is no obligation to take part or answer, the SHIW suffers from a very high non-response rate, but ex-post reweighting is computed in the Survey to account for it (however, this weighting procedure did not help to adjust for missing data or other nonsampling errors related to the income data). Notice that the sum of the survey weights is equal to the total number of sampled units and all household members have the same sample weight.

\textsuperscript{14} Cf. Lugaresi (1989, 1990). The model does not contain estimates of behavioural responses for changes in personal income taxes. Notice that in direct tax models this is rather the rule than the exception.

\textsuperscript{15} See Di Biase et al. (1995, pp. 22-23) and Marenzi (1996) about the methodology.

\textsuperscript{16} According to the stage \(i\), if the empirical researchers were interested to the potential effects of a different tax system, ITAXMOD obtains the potential post-tax income by using directly the algorithm of the tax reform, including all the available information about the attributes of the household and its members.
2005 pre-tax distribution of income, size and scale of inequality parameter values according to a number of isoelastic transformations have been applied on 2000 pre-tax incomes.

To acquire 2005 post-tax income data, all we need is to apply the 2005 IRPEF schedule on each simulated 2005 pre-tax distribution of income.17

Each comparison has, for both points of time, as starting point the two vectors that allow to investigate the potential real redistributive effect of the taxation process, where, for 2005, I simulate the 2005 IRPEF redistributive impact on a range of before tax income distributions all produced by the matching between different isoelastic transformations and the pre-tax distribution of income made available by ITAXMOD00.

3.1 THE PROCEDURE

Let $A = e^a$; according to DL’s Theorem 1, when $g$ is isoelastic:

$$g (x_h) = e^a (x_h)^b$$  \hspace{1cm} (h = 1, 2, \ldots, n).

Nine simulations are presented.18 Several parameter values for $b_i$’s (i = 1,…,9) have been assumed; on the other hand, the value for the size change (the equiproportional grow) is invariant ($e^a = 1.1818$ is the nominal GDP growth rate - plus one - computed between 2000 and 2005).19

17 Between 2000 and 2005, some relevant variations have modified the IRPEF schedule (notice that it is applied on an individual basis): in this paper, I investigate the 2005 tax schedule. Table 1 in the Appendix shows the reduction of the number of fiscal brackets - from five to four - and the variation of the nominal tax rates (the highest was reduced and the lowest was increased). Moreover, with other minor attributes, two new tax allowances have been created. The first one increases the amount of the no-tax area, replacing a few not refundable tax credits and, out from the no-tax area, is decreasing with income, becoming equal to zero for different income level in accordance with different source of income (e.g., lower for self employment, higher for dependent work); the other one replaces all the family size-related tax credits and is decreasing with income. In ISAE pre-tax data are provided earned and self-employment incomes, pensions, entrepreneurial incomes (in the IRPEF tax basis), and other minor incomes sources (notice that incomes from immovable properties are not included). Finally, capital incomes, for the most subject to separate taxation, and fringe benefits are excluded from the IRPEF tax basis.

18 In Russo (ib.), the parameter values capturing the scale of inequality and size variations have been computed by using the weighted least squares (WLS) estimator (I refer to this paper for a discussion about the goodness of these values to show the isoelastic transformations occurrence for the Italian case; see also DL, 2002):

$$\ln (x_h)_{1998} = 0.183490 + 0.990573 \ln (x_h)_{1995} \quad \text{and} \quad \ln (x_h)_{2000} = 0.391951 + 0.976069 \ln (x_h)_{1995} \quad \forall h.$$  

The simulations presented in this paper make use of values for $b$ not so different, but I control also for pre-tax scale of inequality increasing, and, in general, for values more distant from the value ($b = 1$) that does not change the inequality in pre-tax incomes. As noted above, with time series large enough it may be possible to choose the trend analysis approach: I guess that the availability of estimates only for the period 1995-2000 does not allow to make this assertion.

Then, I assume that:

\[(x_h)_{2005} = e^a ((x_h)_{2000})^{b_i} \]  \forall h, \forall i,

or,

\[(x_h)_{2000} = e^{a/b_i} ((x_h)_{2005})^{1/b_i} \quad [= g (x_h)_{2005}] \]  \forall h, \forall i.

Obviously, I choose 2000 as the base-year; for the first four cases \((b_1 = 0.965; b_2 = 0.9775; b_3 = 0.9875; b_4 = 0.995)\), 2005 pre-tax individual microdata display a lower scale of inequality with respect to the base-year; the fifth simulation refers to \(b_5 = 1\) (pre-tax incomes grow equipropotionately in the transition from 2000 to 2005); for the last four cases \((b_6 = 1.005; b_7 = 1.0125; b_8 = 1.0225; b_9 = 1.035)\), a higher - and increasing - pre-tax scale of inequality with respect to the base-year has been assumed. Other comparison outcomes, by assuming values more far from \(b = 1\), can be provided by the author upon request, but I believe they would not be economically significant.

After the 2005 IRPEF schedule application on each simulated 2005 pre-tax distribution of income, some qualifications are required prior to proceed to illustrate the outcomes and to fully explore the pure redistributive effect of tax policies when pre-tax distributional differences are explicitly taken into consideration.

First, household income has to be measured by equivalent income to make it tell about well-being. I choose to adopt a relative equivalence scale for the distribution of household income, both before and after tax, to accommodate for difference in needs.\(^{20}\)

By using a double-parametric function suggested by Cutler and Katz (1992), I deflate each given household money income into units of household equivalent income.\(^{21}\) The deflator \(m\) provides what is named the “number of equivalent adults” in the household and takes the form:

\[m_h = (N_a + \varphi N_e)\theta, \quad 0 \leq \varphi \leq 1, \ 0 \leq \theta \leq 1,\]

---

\(^{20}\) See Ebert and Lambert (2004) about the distinction between relative and absolute equivalence scale.

\(^{21}\) It is well known that levels in measured income inequality can vary depending on the choice of equivalence scale, although none of them has been proved to be superior; there is, hence, a wide agreement about the lack of a unique equivalence scale. Other rules suggested come from Buhmann et al. (1988), Atkinson et al. (1995) and they could be derived also from the Cutler and Katz deflator by the selection of particular parameter values. Decoster-Ooghe (2003) and Creedy-Sleeman (2005) adopt their two-parametric functional form. Finally, for a comparative evaluation with respect to the parametric and econometric approaches to equivalence scales, the interested reader may helpfully look at Cowell-Prats (1999).
where \( N_a \) and \( N_c \) are, respectively, the number of adults and children in the household \( h \), for \( h = 1, 2, \ldots, n \); \( \phi \) is the way in which children are converted in adults and \( \theta \) is the parameter value for economies of scale within the household \( h \). The parameters \( \phi \) and \( \theta \) are assumed to be independent of income.\(^{22}\) Finally, I assume pooled resources within families are distributed equally based on need.

In this application, \( \phi = \theta = 0.5 \) is the value for equivalence scale parameters (notice that there are no coefficient differences between adults, e.g., head versus spouse, or other adults).\(^{23}\) According to the weight recommended by the OECD (1982) in its work on social indicators, the value 0.5 is assigned to each child younger than 14.\(^{24}\)

Thus, each household equivalent income, for both relevant vectors, is derived firstly by collecting income over household members, and then deflating the household monetary income with the relative deflator.

It is worth to highlight that the average utility in the economy is now function of the household equivalent incomes and the household is the unit of analysis (all households, hence, are weighted by 1).

Even by using a conventional equivalence scales transformation to focus on living standards, the likelihood of a horizontally inequitable income tax is very high. When the population is socially homogenous and the only source of difference among people is money income \( x \), this turns out only when the assumption \( 0 \leq T'(x) < 1, \forall x \), is violated.

On the other hand, when the population is not socially homogenous the only relevant differences between households are their money income, sizes and composition. Given the chosen reference type, there is room for horizontal inequity (HI) when, \( \forall h \), the income tax function is not in one of the two following forms (Ebert, 1997; Ebert-Moyes, 2000; Ebert-Lambert, 2004):

\[
\begin{align*}
&i) \quad T(x, h) = m_h \left[ \tau \left( \frac{x}{m_h} \right) \right],
\end{align*}
\]

\(^{22}\) Ebert and Lambert (2004) show that a constant relative scale, \( m(x) = m, \forall x \), meets a pure horizontal equity criterion (that is, pre-tax equals should have the same post-tax living standard) and is (residual) progression preserving.

\(^{23}\) Single adults are the reference type (equivalence scale equal to 1).

\(^{24}\) If \( \theta = 0 \) the scale takes no account of needs, and this value is appropriate if the analyst judges that households’ equivalent income coincides with households’ money income.
where $\tau$ is a tax schedule of the household equivalised income, $\left( \frac{x}{m_h} \right)$, which embodies the degree of vertical equity prescribed by the decision maker, and $m_h$ is the chosen (constant) equivalence scale deflator in accordance with the number of equivalent adults for the household $h$;

\[ ii) \quad T(x, h) = \tau(x - a_h), \]

where $a_h$ is an abatement, according to a given (constant) absolute scale, and $\tau$ is a tax schedule of the household equivalised income $(x - a_h)$.\(^{25}\)

The Italian PIT does not act like the income tax functions just described: as a consequence, several characteristics of the Italian tax system represents potential sources of HI. For instance, the IRPEF operates on the individual pre-tax money income. Furthermore, the Italian exemptions for items of expenditure, other income related deductions, and, as a matter of fact, tax evasion concentrated in particular on self-employment income, can easily cause HI.

Since the aim of this paper is to capture the pure IRPEF redistributive effect, in order to isolate - and exclude - the new inequality introducing by the IRPEF (e.g., within pre-tax income equals groups), the literature provides two prevailing views on how to do.\(^{26}\)

The starting point of the classical HI approach stresses the fact that the before-tax equals have been unequally treated by the taxation: as a consequence, the dispersion of taxes at fixed income levels $x$ comes out.

The no reranking equity criterion refers to HI as a feature of the taxation process, rather than of its outcome. Both approaches lead to different ways to observe the pure vertical stance of a tax schedule.

Without going deeper into the procedures for the classical HI approach\(^{27}\), I choose to adopt the no reranking framework, basically in accordance with the fact that no pre-tax equals are present in micro-data samples.

\(^{25}\) Notice that, respectively, $0 \leq \tau'\left( \frac{x}{m_h} \right) < 1$, $\forall \left( \frac{x}{m_h} \right)$, and $0 \leq \tau'\left( x - a_h \right) < 1$, $\forall \left( x - a_h \right)$.

\(^{26}\) See Marenzi (1995) for a Reynolds-Smolensky index decomposition, showing how much HI – separated into two parts, classical Horizontal Inequity and reranking - is delivered by IRPEF. About the decomposition analysis, see Lambert and Aronson (1993) and Aronson et al. (1994); about classical HI and reranking approaches cf. Lambert (2001).

\(^{27}\) The interested reader may helpfully look at Lambert and Ramos (1997), and Duclos and Lambert (2000).
According to the no reranking approach, vertical equity is about the choice of post-tax equivalent income distribution given the pre-tax distribution. In such a case, Horizontal Equity may be viewed as implying the absence of reranking: there should be perfect association between household pre- and post-tax living standards.  

On the other hand, usually, actual tax systems rerank income units: I must isolate the vertical equity effect from the reranking effect. \( N_i(x) \) should be the post-tax equivalent income whose rank is the same as the pre-tax rank of \( x \), so we need to generate all \( N_i(x), i = (2000, 2005) \), sample post-tax distributions from the sample pre-tax distributions.

The only way to construct such a function is to sort separately the pre- and post-tax equivalent income distribution in each sample: it has to break the disassociation, if any, which is present.

Each \( N_i(x) \) still maps existing pre-tax living standards to existing post-tax living standards, but in a different order: they enjoy now perfect and positive association. There are no effects on post-tax inequality because the only variation is the rank of each household. Thus, the sorting procedure is inequality neutral.  

The last stage before going on is to assure that no selectivity bias is present into the sample. Survey weights have been assigned to each sample case by the SHIW of the Bank of Italy. To each household is attached a sample weight in inverse relation to its probability to be included into the sample, and the procedure adopted here considers the weight structure in the sample design following the standard proceeding, i.e. to take into account how much each observation in the data set influences the final estimates.

Finally, outlined samples are truncated to eliminate observations reporting zero incomes, and the top 0.5% from each sample are removed to limit the dependency of results on outliers: I have now at hand, for both period, HI-free pre- and equal yield post-tax equivalent incomes useful to evaluate and compare with respect to the 2000 tax scheme, the potential and pure real redistributive effect of the 2005 IRPEF reform.

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29 See King (1983), DL (2002). However, the progressive stance obtained by sorting pre- and post-tax distributions reveals slight differences with respect to the alternative procedure characterizing the classical HI approach (see Dardanoni and Lambert, 2001, on this). Notice that the sorting procedure should constrain the practitioner to make use of the remark 2.1 in Ebert-Moyes (2002), if she or he wishes to use distinct weighting schemes consistent with different income recipient (cf. Russo, 2006).
30 See Russo (2005, p. 239) for an alternative weighting procedure.
31 Cf. footnote 9, ivi, on this. However, notice that in this paper - as in the standard methodology - it has to no take into account population structure and composition change over time.
4. THE RESULTS

To give a rough idea of the redistributive story, Table 2 shows the Reynolds-Smolensky redistributive effect indexes for 2000 and 2005 (for $i = 1,\ldots,9$).

**TABLE 2 - Reynolds-Smolensky Indexes**

<table>
<thead>
<tr>
<th>$b_i$</th>
<th>$\Pi_{RS}^{2000}$</th>
<th>$\Pi_{RS}^{2005}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.965</td>
<td>0.037616</td>
<td>0.035361</td>
</tr>
<tr>
<td>0.9775</td>
<td>0.037606</td>
<td>0.035361</td>
</tr>
<tr>
<td>0.9875</td>
<td>0.038742</td>
<td>0.039358</td>
</tr>
<tr>
<td>0.995</td>
<td>0.039709</td>
<td>0.040027</td>
</tr>
<tr>
<td>1</td>
<td>0.040374</td>
<td>0.040408</td>
</tr>
<tr>
<td>1.005</td>
<td>0.040408</td>
<td>0.040408</td>
</tr>
<tr>
<td>1.0125</td>
<td>0.040729</td>
<td>0.040974</td>
</tr>
<tr>
<td>1.0225</td>
<td>0.04125</td>
<td>0.04125</td>
</tr>
<tr>
<td>1.035</td>
<td>0.04173</td>
<td>0.04173</td>
</tr>
</tbody>
</table>

The effective progression outcomes don’t offer an unambiguous answer: when the change in the pre-tax distribution affects only the size ($b = 1$), $\Pi_{RS}^{2005}$ is higher with respect to $\Pi_{RS}^{2000}$; when the scale of inequality decreases, the index value starts to reduce and, for $b = 0.9775$, it becomes lower with respect to the benchmark index value for 2000, achieving the minimum value for $b = 0.965$ (notice that this value represents a very significant increase of 2005 pre-tax scale equality with respect to the base-year).

On the other hand, for parameter values capturing an higher pre-tax disparity with respect to the base-year, $\Pi_{RS}^{2005}$ is always superior with respect to $\Pi_{RS}^{2000}$, it slightly increases and reaches the peak when $b = 1.035$.

To obtain conclusive assessments about the second step of Italian PIT reform, I provide comparisons based on the criterion of Lorenz dominance. The usual Lorenz curve diagram is not presented, because it provides scarce and not fully representative information; in order to be more accurate about the actual redistributive findings, I prefer to present figures with the
difference between the gaps among post- and pre-tax household equivalent income cumulated shares. For the generic comparison:

\[
[L_N^{2005}(p) - L_X^{2005}(p)] - [L_N^{2000}(p) - L_X^{2000}(p)] \geq 0 \quad \forall p, \text{ with } > \text{ for some } p.
\]

Figures 1(a) and 1(b) plot, i.e., 2005 versus 2000, \( \forall i \), denoting the empirical evidence with regard to the difference between the 2005 post- and pre-tax Lorenz curve gap and the corresponding gap for 2000. The absence of negative values for all \( p \), and positive values for some \( p \), means Lorenz dominance of 2005 (i.e., \( \langle N_{2005}, F_{2005} \rangle \phi_L \langle N_{2000}, F_{2000} \rangle \)). The opposite case means 2000 dominance.
As easily observed, 2000 redistribution Lorenz dominates over 2005 for $b = 0.965$. However, this characteristic is quickly lost. By increasing the 2005 pre-tax scale of inequality, the direction of the variation of the pure and real 2005 redistributive effect seems to be apparent, it is always higher, and 2005 redistribution becomes to dominate over 2000 for $b = 1$. In intermediate cases Lorenz curve intersections are occurred, hence Atkinson’s theorem is not helpful.

FIGURE 1(b)
In accordance with figure 1(b), the larger is the 2005 pre-tax disparity, the greater is the evidence for 2005 redistribution dominance over 2000, and this result appears to be always more apparent.\(^{32}\)

Reynolds-Smolensky indexes outcomes cannot be taken as informative with regard to the normative assessment for \(b = 0.9775\), \(b = 0.9875\), and \(b = 0.995\): for estimated Lorenz curves crossing cases this progressivity measure should be considered a descriptive index.\(^{33}\)

Now, it may be appealing to test the already referred Musgrave and Thin (ib.) expectation, “[…] the less equal the distribution of income before tax, the more potent will be a (given) progressive tax in equalizing income”.\(^{34}\)

Furthermore, it may be interesting to verify suggestions coming out from the figures 1(a) and 1(b): is the evidence with regard to an increase with the scale of inequality of pre-tax incomes 2005 IRPEF redistributive power untruthful?

\(^{32}\) For practitioners paying attention to the JK’s result, it is straightforward to make use of the DL’s procedure and, hence, search for actual local-to-global comparison (see the Appendix B). This exercise has been done and outcomes can be provided by the author upon request; the 2000 tax system strongly mimics the pattern of 2005 and it is rather complicated to qualify the trend of \(RP\) elasticities differentials between 2000 and 2005. For intertemporal comparisons of tax regimes, log transplant curves still continue to provide scarce information (see Russo, 2005) and only if a very relevant change of the pre-tax distribution (that is, for \(b = 0.965\) and \(b = 1.035\)) were at work, would this tool become a bit helpful (in contrast, see DL (ib.) for an application where log transplant curves are useful for a local-to-global intertemporal comparison).

\(^{33}\) In this version of the paper, the incidence of the prices variation occurred over the period between 2000 and 2005 is not examined. Also Baldini et al. (2006) pay attention to the real redistributive effects: they highlight the fact that, over the period between 2001 and 2005, the negative redistributive effect of inflation affects the entire range of equivalent income parade; as a consequence the 2005 IRPEF redistributive impact should be undermined.

\(^{34}\) See Moyes (1989) and Lambert-Pfähler (1992) about theoretical conditions we need for the accuracy of the prediction and some particular cases which accomplish it.
It is straightforward to make use of the method presented here to furnish an answer to this question: actually, I proposed 2005 post-tax distributions of income all derived from the same tax schedule and different, with respect to the scale of inequality, pre-tax distributions of income.

The Reynolds-Smolensky index is a measure able to confirm the Musgrave and Thin’s conjecture: given the benchmark defined by the minor parameter value, \( b = 0.965 \), it is well thought-out if the \( \Pi_{2005}^{RS} \) index corresponding to each contiguous superior value is always higher (for instance, 0.965 vs. 0.9775; 0.9775 vs. 0.985; etc).

In more simple terms, all we need is to search for an increasing with \( b \Pi_{2005}^{RS} \) index value.

Table 2 values would allow us to affirm the Musgrave and Thin (ib.) expectation as true: the higher the pre-tax scale of inequality, the more redistributive appears to be the IRPEF reform.

Figure 2 takes up again this matter, by describing the association between each \( \Pi_{2005}^{RS} \) and the corresponding parameter \( b \).

**FIGURE 2**

![Musgrave-Thin's Test graph]

Notice that, for \( b = 0.9775 \), the \( \Pi_{2005}^{RS} \) measure (= 0.037606) is really close to the measure for \( \Pi_{2000}^{RS} \) (= 0.037616).
In such a case, it does make it sense to retain the parameter value 0.9775 as a very good proxy for the value which would indicate the substantial scale of pre-tax inequality reduction by which the 2005 IRPEF reform achieves, more or less, the same real redistributive impact with respect to the 2000 IRPEF. Given the progressivity measure adopted, when the parameter $b$ is higher than 0.9775 the 2005 IRPEF schedule begins to be more redistributive.

To obtain an unambiguous evaluation about 2005 Italian PIT reform, figure 3 illustrates the Lorenz quasi-ordering: the behaviour of the redistributive impact is investigated by verifying a sequential ordering.

With respect to the Italian case, it has to consider the Musgrave and Thin’s prediction as entirely accurate if, given the benchmark defined by the minor parameter value, $b = 0.965$, the redistribution acted by the 2005 IRPEF on the pre-tax distribution of income is, $\forall i$, strictly increasing with $b$.

The symbol “$|\,$” means “conditioned by”; thus, $\forall i$, it has to be:


$$\rightarrow \left(\frac{L^{2005}_N - L^{2005}_X}{L^{2005}_N - L^{2000}_X}\right)_{b_1} - \left(\frac{L^{2005}_N - L^{2005}_X}{L^{2005}_N - L^{2000}_X}\right)_{b_{1+1}} \leq 0 \quad \forall p, \text{ with } > \text{ for some } p.$$
In accordance with figure 3, by using the Lorenz dominance criteria no unambiguous welfare significance can be asserted for all comparisons. However, for six comparisons the 2005 post-tax distribution of equivalent incomes related to the more unequal 2005 pre-tax distribution is always less unequal. Notice that, for these pairwise comparisons, the higher is the pre-tax scale of inequality, the less significant appears to be the dominance.

The Lorenz curve shapes are informative. The higher is the pre-tax scale of inequality, the higher is the number of observations in the right tail of the simulated 2005 pre-tax distribution, and, for the last two comparisons, it turns out an intersection from below. At the top range of the income parade, the progressivity gain is higher when the 2005 IRPEF is applied on the “close” more equal simulated 2005 pre-tax distribution. It follows that, for the scale of pre-tax inequality parameter values adopted, the Tremonti’s reform redistributive power reaches the maximum progressivity in the Lorenz sense when the parameter $b$ is equal to 1.0125.

4.1 CONCLUSIONS

Empirical researchers wishing to assess the outcomes of a tax reform frequently deal with the fact that large-scale micro-data national surveys are available with some delay with respect to practitioner’s needs. Moreover, a tax reform proposal often will act on an unknown and coming pre-tax income distribution. On the other hand, according to the fact that the income tax redistributive effect is mainly determined by the matching between the pre-tax income distribution and the tax schedule, if the practitioner’s aim is

- to control the outcome of standard analysis with regard to pure redistributive effect of a tax reform and
- to avoid any baseline dependence controversy,

she or he should take into account pre-tax distributional differences.

In order to reach both purposes, this paper proposes which should be her/his first task, that is, to investigate if there is a substantial stability of the structure of pre-tax income distributions on her/his hand. According to DL’s procedure (2002), she or he should look for an isoelastic transformation between these pre-tax distributions. If this were the case, I argue that if an isoelastic transformation does hold between pre-tax distributions through time, it seems reasonable enough to conclude that this will occur again.

As a consequence, a more comprehensive approach to analyse the redistributive impact of tax reforms comes out, and, in this paper, a definition of this new procedure has been fully
provided. By controlling for a range of economically realistic potential pre-tax distributional variations, the standard analysis based on a common pre-tax income distribution may produce more robust results. According to a sensitivity analysis approach, in order to simulate different unknown pre-tax income distributions, different isoelastic transformations – i.e., different size and scale of inequality parameter values - should be applied on the most recent available pre-tax distribution of income. To acquire the post-tax income data, all we need is to apply the tax reform schedule on each simulated pre-tax distribution of incomes.

As far as I know, on the basis of potential and actual (pure) redistributive effects intertemporal comparisons of tax systems have not been treated elsewhere before.

By using the sensitivity analysis approach, a first application has been presented for the Italian case. In Italy, it appears that 1995, 1998, and 2000 pre-tax distributions differ basically only by location and scale of inequality, and the isoelastic transformations occurrence allows us to claim for the stability of the structure of Italian pre-tax income distributions. Notice that the main assumption is the assertion for the permanence of a “stable society” with respect to the incomes and income distributions of the country (they should continue to develop gradually).

Even if in this paper this method is used to compare different PIT systems redistributive power, the reader should pay attention to the fact that simulated pre- and post-tax distributions can be useful to derive valuable information also on potential trends in Horizontal inequity, according both to the classical approach and the no reranking procedure.

Obviously, more work needs to be done: firstly, the insights I have gained in this paper should be compared with the results of the standard procedure based on a common distribution of pre-tax income for all schedules being compared.

It is worth to note that outcomes could be different in accordance with different equivalence scale parameters and, in particular, weighting schemes: in order to reflect the preferred normative principle, distinct definitions of income and/or income recipient should be advocated for the Lorenz Criterion (this is a topic for future empirical research; indeed, to assign to each household the same weight implies that income distribution comparisons appear to have little rationale). 35

35 About the normative significance of some possible alternative combinations, the interested reader may refer to, among others, Shorrock's (1995), Ebert (1999) and Ebert-Moyes (2002); see also Cowell (1984). On researches about the dependence of empirical results on various alternatives, cf. Decoster-Ooghe (ib.) and Creedy-Sleeman (ib.).
With regard to the main results of the Italian case, the paper shows that the variation in the pure redistributive impact of the Italian PIT between 2000 and 2005 is not so remarkable; the alternative way to present the comparison of Lorenz curves demonstrates that potential redistributive effect differences were small (e.g., the highest redistributive effect gain is around 0.003) and their economical significance low.

However, according to equivalence scale parameters (and for practitioners persuaded that a Social Welfare Function coherent with the Pareto-criteria and inequality-adverse is related with the actual well-being of the society), the current analysis has determined a welfare ranking according to Atkinson’s theorem for the majority of the Lorenz comparisons. The results are to some extent sensitive to the choice of the parameter of the scale of pre-tax inequality. When the parameter embodies the lowest degree of inequality \((b = 0.965)\), 2000 redistribution dominates over 2005; on the other hand, when \(b\) capture a higher degree of pre-tax inequality with respect to 2000 \((b = 1, 1.005, 1.0125, 1.0225, 1.035)\), 2005 redistribution dominates over 2000. Notice that when the change in the pre-tax distribution affects only the size \((b = 1)\), at the bottom range of the income parade the 2005 redistributive gain with respect to 2000 is substantially irrelevant. For the other three cases, estimated Lorenz curves cross and it is not possible to infer normative outcomes.\(^{36}\)

Finally, what does figure 3 tell us about Musgrave and Thin prediction (\(ib\).)? It is not generally possible to accept it, because the influence, \(ceteris paribus\), of changes of the pre-tax distribution of incomes is ambiguous. Starting from the lowest degree of scale of pre-tax inequality \((b = 0.965)\), the pure redistributive effect of 2005 IRPEF is initially increasing with \(b\), but the higher is the pre-tax scale of inequality, the less significant is the dominance, and for the last two cases, it turns out an intersection: as a consequence, the redistributive power of the 2005 IRPEF reaches the maximum progressivity when the sensitivity parameter is equal to 1.0125.

Clearly, a fully convincing actual redistributive effect analysis with respect to the 2005 IRPEF versus the 2000 IRPEF has to be based also on the actual 2005 before tax distribution of incomes. In the mean time, we shall have the best test to empirically corroborate the goodness of the innovative procedure here defined. We need to wait for.

\(^{36}\) With regard to the issue of performing statistical inference for the Lorenz quasi-ordering the interested reader may refer to Dardanoni and Forcina (1999), Davidson and Duclos (1997, 2000), and Barret and Donald (2003). This paper does not present tests for equality of two empirical Lorenz curves, or tests for Lorenz dominance between two curves (notice that a sample crossing excludes the existence of a population ordering). I stress that statistical significance is relevant in particular when there is empirical evidence of economically significant dominance (the same holds for \(\Pi^{RS}\) indexes). I believe that stylized facts above displayed do not allow stating this claim for the 2005 IRPEF versus the 2000 tax scheme.
REFERENCES


APPENDIX A – TABLE APPENDIX (*)

TABLE 1

(a) - 2000 IRPEF

<table>
<thead>
<tr>
<th>brackets of taxable income</th>
<th>tax rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10,329.138</td>
<td>18.5%</td>
</tr>
<tr>
<td>10,329.138 – 15,493.707</td>
<td>25.5%</td>
</tr>
<tr>
<td>15,493.707 – 30,987.414</td>
<td>35.5%</td>
</tr>
<tr>
<td>30,987.414 – 69,721.6814</td>
<td>39.5%</td>
</tr>
<tr>
<td>Over 69,721.68</td>
<td>45.5%</td>
</tr>
</tbody>
</table>

(b) - 2005 IRPEF

<table>
<thead>
<tr>
<th>brackets of taxable income</th>
<th>tax rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 26,000</td>
<td>23%</td>
</tr>
<tr>
<td>26,000 - 33,500</td>
<td>33%</td>
</tr>
<tr>
<td>33,500 - 100,000</td>
<td>39%</td>
</tr>
<tr>
<td>Over 100,000</td>
<td>43%</td>
</tr>
</tbody>
</table>

(*) Excluding the Table 1, both Tables and Figures are produced by author’s calculations (MATLAB 6.5 is the software package). Values in Euro.
In accordance with, for instance, part b) of Theorem 2 (DL, 2002) the identification of the coefficients $A$ and $b$ is called for. There needs to find the values for size and scale of inequality defining the isoelastic transformations of pre-tax distribution in equivalent terms (notice that the 2000 and 2005 pre- and post-tax distributions of income are initially related to a sample of individuals). The WLS estimator appears to be the simplest way to derive the constant and slope in a regression of, for the generic comparison, $\ln(x_h)_{2000}$ on $\ln(x_h)_{2005}$; hence, now in equivalent terms,

$$\ln(x_h)_{2005} = a + b \ln(x_h)_{2000}, \text{ or } \ln(x_h)_{2000} = (-\frac{a}{b-1}) + \left(\frac{1}{b-1}\right) \ln(x_h)_{2005} \forall h, \forall i.$$  

If the practitioner is ready to accept that the distribution of 2000 log pre-tax equivalent incomes is a good proxy for the distribution of 2005 log pre-tax equivalent incomes corrected for distributional differences, now we need to correct for the pre-tax distributional differences acting on log post-tax equivalent income distributions, i.e. before and after the tax reform. Following DL (ib.), with 2000 as base-year, we should correct 2005 log post-tax equivalent income distributions; for the generic comparison:

$$\langle N_{2005}, F_{2005} \rangle^g = \langle N_{2005}^g, F_{2005} \circ g^{-1} \rangle,$$

where $N_{2005}^g = g \circ N_{2005} \circ g^{-1}$. By assumption, it is $g^{-1} = F_{2005}^{-1} \circ F_{2000}$, hence,

$$\langle N_{2005}, F_{2005} \rangle^g = \langle N_{2005}^g, F_{2000} \rangle.$$

It has to compare this regime with

$$\langle N_{2000}, F_{2000} \rangle.$$

It can be shown that, for $i = 1, \ldots, 9$:

$$\ln N_{2005}^g (x_h) = (-\frac{a}{b-1}) + \left(\frac{1}{b-1}\right) \ln N_{2005} (x_h) \forall h.$$  

In order to compare residual progression measures, a suitable way is to examine the log transplant curve slopes, hence, $\forall i$, to plot in the same diagram,

(i) $\ln(x_h)_{2000}$ vs. $\ln N(x_h)_{2000}$ $\forall h$, and

(ii) $\ln(x_h)_{2000}$ vs. $(-\frac{a}{b-1}) + \left(\frac{1}{b-1}\right) \ln N_{2005} (x_h) \forall h.$

The isoelasticity transformation for equivalent pre-tax incomes would allow us to consider, in regard to the $x$-axis for each of the nine log transplant curve diagrams, the distribution of 2000 log pre-tax equivalent income alone - that is, if we are ready to accept that it is a good proxy for the distribution of 2005 log pre-tax equivalent income corrected for distributional differences. The conditions for the JK’s theorem are respected and residual progression elasticities for each corrected 2005 log post-tax equivalent income distribution would be, of course, the actual ones. Finally, notice that the starting point of the analysis is usually to deflate (or reflate) income time series to avoid the effect of inflation: by assuming isoelasticity, it is possible to skip this phase because there is no need to convert nominal values into real values before applying the transplant-and-compare procedure (DL, ib., p. 111, footnote 19).