THE TAXI MARKET: FAILURES AND REGULATION

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Abstract

We consider a model of the taxi market. The demand for taxi-trips is a function of both the price and the number of vacant taxi, i.e. the supply side. Similarly, the supply is a function of both the price and the number of waiting passengers, i.e. the demand side. We characterize the social optimum and evidence that the corresponding allocation would not allow the industry to break-even. We show that in the case of the competitive market, prices are above the social optimum and vacancies below. The latter is because a competitive market fails to take into account the effects of vacant taxis on demand. We also show that, in the case of a monopoly, despite the complex interactions on the market, the profit maximising price obeys the standard Lerner formula. However, while a marginal increase in the number of vacant taxi would have no effect on firm’s profits, it has a strictly positive value to the consumers. In other words, the monopoly is unable to extract the whole willingness to pay of the consumers. As in the competitive case, the fleet size of a profit-maximising monopolist is always below the social optimum level, although to a less extent. In term of prices, the second-best allocation is characterised by the standard Ramsey formula. We show that there is strictly less distortion at second best than in the competitive market. This provides a rational for regulation. Given that price control would not be sufficient to reach the second-best allocation, we propose to decentralise the allocation through an extended price-cap scheme. The latter requires the regulator to know the average value of time and an estimate of the marginal impact of an additional taxi in the fleet on the expected queuing time.

Keywords: taxi; regulation, generalised price-cap

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1 The model

There are three variables of interest:

- $X$, the number of trips (taxi actually matched with a passenger)
- $V$, the number of vacant tax (waiting for passengers)
- $W$, the number of passengers waiting (for a taxi)

The utility derived from $X$ trips writes $U(X)$
The costs of providing $X$ trips writes $C(X)$
The cost of having $V$ vacant taxi writes $C(V)$
The cost of waiting (disutility) of the $W$ passengers writes $C(W)$
The “matching function” writes

$$m = f(X, V, W),$$

with $(\partial f / \partial X) < 0$, and $(\partial f / \partial V) > 0$ and both $(\partial f / \partial W) > 0$.

Clearly at equilibrium (stationary state, since all values are per unit of time) $m = f(X, V, W) = X$. It follows that one can rewrite

$$X = g(V, W)$$

with

$$\left( \frac{\partial g}{\partial V} \right) = \frac{(\partial f / \partial V)}{1 - (\partial f / \partial X)} > 0$$
$$\left( \frac{\partial g}{\partial W} \right) = \frac{(\partial f / \partial W)}{1 - (\partial f / \partial X)} > 0.$$

2 Behaviour of the travellers (demand)

The total demand for taxi trip is given by the sum of occupied taxis and the number of waiting passengers ($X + W$) and generate a net utility:

$$U(X) - \left[ pX + \tilde{C}(W) \right]$$

where $pX + \tilde{C}(W)$ is the generalised cost of service (or full price).

Assuming a representative consumer, her decision variable is $W$. It follows that:

$$[U'(X) - p] g_W - \tilde{C}'(W) = 0.$$

with the standard notation $g_W = (\partial g / \partial W)$. 

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Hence, at equilibrium, the traveller marginal utility must be equal to the marginal generalised cost of $X$:

$$\bar{p} = U'(X) = p + \frac{1}{g_W} \tilde{C}'(W).$$

(3)

The second component of the right hand side of (3) is the marginal increase in the total cost of waiting generated by an additional passenger ($\tilde{C}'(W)$), corrected for the effect on total utility of the increase in the number of matches induced by the new demand ($g_W$).

Observe that $W$ is a function of both $p$ and $V$. Its properties can be obtained by deriving equation (2) with respect to both variables:

$$U''(X) g_W \frac{\partial W}{\partial p} = 1 + \frac{d}{dW} \left( \frac{1}{g_W} \tilde{C}'(W) \right) \frac{\partial W}{\partial p}$$

$$U''(X) \left[ g_V + g_W \frac{\partial W}{\partial V} \right] = - \frac{g_V W}{(g_W)^2} \tilde{C}'(W) + \frac{\partial}{\partial W} \left( \frac{1}{g_W} \tilde{C}'(W) \right) \frac{\partial W}{\partial V}$$

to obtain:

$$\frac{\partial W}{\partial p} = - \left[ \frac{d}{dW} \left( \frac{1}{g_W} \tilde{C}'(W) \right) - U''(X) g_W \right]^{-1}$$

(4)

$$\frac{\partial W}{\partial V} = \frac{\frac{g_V W}{(g_W)^2} \tilde{C}'(W) + U''(X) g_V}{\frac{d}{dW} \left( \frac{1}{g_W} \tilde{C}'(W) \right) - U''(X) g_W}.$$  

(5)

We will assume $\frac{\partial W}{\partial p} < 0$ which, with a quasi-concave utility function, implies $\frac{d}{dW} \left( \frac{1}{g_W} \tilde{C}'(W) \right) > 0$; and that $\frac{\partial W}{\partial V} > 0$, which implies, in addition, that $\frac{g_V W}{(g_W)^2} \tilde{C}'(W) > |U''(X) g_W| > 0$.

Note that the marginal impact of the number of vacant taxi $V$ on the traveler net utility is:

$$\frac{\partial}{\partial V} \left( U'(X) - \left[pX + \tilde{C}(W)\right] \right) = (U'(X) - p) \frac{\partial X}{\partial V} - \tilde{C}'(W) \frac{\partial W}{\partial V}$$

$$= (U'(X) - p) \left( g_V + g_W \frac{\partial W}{\partial V} \right) - \tilde{C}'(W) \frac{\partial W}{\partial V}$$

$$= \frac{1}{g_W} \tilde{C}'(W) \left( g_V + g_W \frac{\partial W}{\partial V} \right) - \tilde{C}'(W) \frac{\partial W}{\partial V}$$

$$= \frac{g_V}{g_W} \tilde{C}'(W).$$

(6)
The marginal utility of $V$ is thus given by the saving, at the margin, in aggregate disutility from waiting ($\frac{\tilde{C}'(W)}{g_W}$) induced by the increase in matches ($g_v$).

Similarly, the marginal impact of the fare $p$ on the traveler net utility is:

$$\frac{\partial}{\partial p} \left( U(X) - \left[pX + \tilde{C}(W)\right]\right) = (U'(X) - p) \frac{\partial X}{\partial p} - \tilde{C}'(W) \frac{\partial W}{\partial p} - X$$

$$= \left[(U'(X) - p) g_W - \tilde{C}'(W)\right] \frac{\partial W}{\partial p} - X$$

$$= -X$$

which is standard.

3 Social optimum

The social welfare function writes:

$$SW(X, V, W) = U(X) - C(X) - \tilde{C}(V) - \tilde{C}(W), \quad (7)$$

where $X = g(V, W)$

The FOC of the social welfare maximisation problem (first best) are:

$$\frac{dSW^{FB}}{dV} = g_v [U'(X) - C'(X)] - \tilde{C}'(V) = 0 \quad (8)$$

$$\frac{dSW^{FB}}{dW} = g_w [U'(X) - C'(X)] - \tilde{C}'(W) = 0. \quad (9)$$

Thus,

$$U'(X) - C'(X) = \frac{\tilde{C}'(V)}{g_v} = \frac{\tilde{C}'(W)}{g_w} > 0. \quad (10)$$

Observe that $U'(X) > C'(X)$ although social optimum still follows from marginal cost pricing. More precisely, if $p = C'(X)$, the demand equation, as characterised by (2), gives:

$$p = U'(X) - \frac{1}{g_w} \tilde{C}'(W) = C'(X). \quad (11)$$

If, in addition, we have

$$\tilde{C}'(V) = \frac{g_v}{g_w} \tilde{C}'(W), \quad (12)$$

then (10) holds true.

The right hand side of (12) is the marginal impact of the number of vacant taxis on consumer net utility (see equation (6)). Thus while equation (11) states that the monetary costs to the traveller of an additional trip $X$ should be tarifed at its marginal cost, equation (12) states that the time benefits to the traveller of an additional taxi $V$ should also be equal to its marginal cost.
3.1 Profits at the social optimum:

Profits are:

$$\Pi = pX - C(X) - \tilde{C}(V).$$  \hspace{1cm} (13)

Combining (3) with (10), at social optimum we have:

$$p - C'(X) = \frac{1}{gV} \tilde{C}'(V) - \frac{1}{gW} \tilde{C}'(W) = 0.$$  

Hence, with constant returns to scale ($C_0(X) = \frac{C(X)}{X}$),

$$\Pi^{FB} = -\tilde{C}(V),$$

at the social optimum, profits are negative for an amount equal to the cost of vacancies. This is a well-known result in the literature (see for instance, Arnott (1996) and Cairns and Liston-Heyes (1996)): because of the social value of the unused capacity (vacancies) at social optimum the taxi industry should be subsidised.

The marginal profits of an increase in $p$ and $V$ write respectively:

$$\frac{\partial \Pi}{\partial p} = X + (p - C'(X)) \frac{gW}{p} \frac{\partial W}{\partial p}$$  \hspace{1cm} (14)

and

$$\frac{\partial \Pi}{\partial V} = (p - C'(X)) \left( gV + gW \frac{\partial W}{\partial V} \right) - \tilde{C}'(V).$$  \hspace{1cm} (15)

Since at the social optimum $p = C'(X)$, it follows that:

$$\frac{\partial \Pi^{FB}}{\partial p} = X,$$  \hspace{1cm} (16)

$$\frac{\partial \Pi^{FB}}{\partial V} = -\tilde{C}'(V).$$  \hspace{1cm} (17)
4 Behaviour of the firm on a competitive market

In a competitive market a single firm would be price taker and "waiting passenger taker". The latter since it considers that its decision on $V$ (entering the market) has no impact on $W$ (the demand for taxi trips).

For given $p$ and $W$, the firm maximises profits ($\pi^C$), as given by (13), in $V$:

$$\frac{\partial \pi^C}{\partial V} = [p - C'(X)] g_V - \tilde{C}'(V) = 0,$$

Equation (2) and (18) determine demand and supply function respectively. In equilibrium we have:

$$p = U'(X) - \frac{1}{g_W} \tilde{C}'(W) = C'(X) + \frac{1}{g_V} \tilde{C}'(V) \tag{19}$$

where $X = g(V,W)$.

It follows in particular that

$$U'(X) - C'(X) = \frac{1}{g_V} \tilde{C}'(V) + \frac{1}{g_W} \tilde{C}'(W) > 0. \tag{20}$$

Substituting (19) in (14) and in (15), we obtain the marginal profit deriving, respectively, from an increase in $p$ and in $V$:

$$\frac{\partial \Pi^C}{\partial p} = X + \frac{g_W}{g_V} \tilde{C}'(V) \frac{\partial W}{\partial p} < X \tag{21}$$

$$\frac{\partial \Pi^C}{\partial V} = \frac{g_W}{g_V} \tilde{C}'(V) \frac{\partial W}{\partial V}. \tag{22}$$

From the latter equation we can observe that a higher number of taxis would actually increase profits: this is because a competitive market fails to take into account the effect of the number of vacant taxis on demand, which is clear comparing (18) with (15).
5 Behaviour of the monopolist

Let’s consider the behaviour of a monopolist. The FOC of the profit-maximisation problem are:

\[
\frac{\partial \Pi^M}{\partial p} = X + [p - C'(X)] g_W \frac{\partial W}{\partial p} = 0
\]  
(23)

\[
\frac{\partial \Pi^M}{\partial V} = [p - C'(X)] (g_V + g_W \frac{\partial W}{\partial V}) - \tilde{C}'(V) = 0
\]  
(24)

Note that:

\[
\frac{\partial X}{\partial p} = g_W \frac{\partial W}{\partial p},
\]

hence, from equation (23), one may derive the usual Lerner formula:

\[
\frac{p - C'(X)}{p} = \frac{1}{\varepsilon_X},
\]  
(25)

where

\[
\varepsilon_X = \frac{p}{X} \left( \frac{-\partial X}{\partial p} \right).
\]  
(26)

By using equation (2) describing travellers demand, one may rewrite equation (24) as

\[
U'(X) - C'(X) = \frac{1}{g_V + g_W \frac{\partial W}{\partial V}} \tilde{C}'(V) + \frac{1}{g_W} \tilde{C}'(W),
\]  
(27)

While equation (25) is to be compared with the price at social optimum as defined by (11) and the price on a competitive market as defined by (19), equation (27) is to be compared with (10) and (20), respectively for the social optimum and the competitive market allocation. Interestingly enough, the later comparison shows that, in terms of vacancies, there is less distortion when there is a profit-maximising monopolist rather than a competitive market situation. This follows from the fact that the profit-maximising monopolist is able to take into account the positive externalities vacancies exert on demand, while we assume that firms do not so on a competitive market. Equations (27) and (20) state that, ceteris paribus (i.e. if prices were identical for a profit maximising monopolist and on a competitive market), a profit-maximising would offer more taxi than what would emerge on a competitive market. However, while prices a priori differ, it does not need to be so at equilibrium.

6 Second-best

Let’s turn to the second best solution which consists in maximising social welfare subject to the industry’s break-even constraint. Let \( L \) be the Lagrangian expression associated with this problem, while \( \lambda \) is the multiplier of the break-even constraint:
\[ L = U(X) - C(X) - \tilde{C}(V) - \tilde{C}(W) + \lambda \left[pX - C(X) - \tilde{C}(V)\right] \]

The FOC which characterise the constrained optimal prices are:

\[
\frac{\partial L}{\partial \rho} = \left[U'(X) - C'(X)\right] \frac{\partial X}{\partial \rho} - \tilde{C}'(W) \frac{\partial W}{\partial \rho} + \lambda \left[X + (p - C'(X)) \frac{\partial X}{\partial \rho}\right] = 0, \quad (28)
\]

\[
\frac{\partial L}{\partial V} = \left[U'(X) - C'(X)\right] \frac{\partial X}{\partial V} - \tilde{C}'(V) - \tilde{C}'(W) \frac{\partial W}{\partial V} + \lambda \left[p - C'(X)\right] \frac{\partial X}{\partial V} - \tilde{C}'(V) \quad (29)
\]

where:

\[
\frac{\partial X}{\partial V} = gV + gw \frac{\partial W}{\partial V}. \quad (30)
\]

From (3) the FOC (28) rewrites:

\[
\lambda X + (1 + \lambda) (p - C'(X)) \frac{\partial X}{\partial \rho} + \tilde{C}'(W) \left[\frac{1}{gV} \frac{\partial X}{\partial \rho} - \frac{\partial W}{\partial \rho}\right] = 0,
\]

that simplifies to:

\[
\lambda X + (1 + \lambda) (p - C'(X)) \frac{\partial X}{\partial \rho} = 0, \quad (31)
\]

which rewrites as the usual Ramsey formula

\[
\frac{p - C'(X)}{p} = \frac{\lambda}{1 + \lambda \varepsilon_X}. \quad (32)
\]

Similarly, one can rewrite the FOC (29) by using the demand equation (2) to obtain:

\[
U'(X) - C'(X) = \frac{gV}{gV + gw} \tilde{C}'(V) + \left(\frac{\lambda \varepsilon_X gV + gw \frac{\partial W}{\partial V}}{gV + gw \frac{\partial W}{\partial V}}\right) \tilde{C}'(W). \quad (33)
\]

Comparison of the later equation with (27) is to be compared with (10) and (20)

6.1 Effects of marginal changes

We know that:

\[
\frac{\partial X}{\partial \rho} = gW \frac{\partial W}{\partial \rho}
\]
Substituting in (31) we find:

\[(p - C'(X))g_W \frac{\partial W}{\partial p} = -\frac{\lambda}{1 + \lambda}X.\]  

(34)

Substituting in (14) we have:

\[\frac{\partial \Pi^{SB}}{\partial p} = X - \frac{\lambda}{1 + \lambda}X.\]

(35)

Similarly, from (3) the FOC (29) rewrites:

\[0 = (1 + \lambda) \left( (p - C'(X)) \frac{\partial X}{\partial V} - \tilde{C}'(V) \right) + \tilde{C}'(W) \left[ \frac{1}{g_W} \frac{\partial X}{\partial V} - \frac{\partial W}{\partial V} \right].\]

From (30), the latter equation rewrites:

\[\frac{\tilde{C}'(V)}{g_V} = \frac{1}{1 + \lambda} \frac{\tilde{C}'(W)}{g_W} + (p - C'(X)) \left( 1 + \frac{g_W}{g_V} \frac{\partial W}{\partial V} \right).\]  

(36)

Substituting in (15) rewrites:

\[\frac{\partial \Pi^{SB}}{\partial V} = -\frac{1}{1 + \lambda} \frac{g_V}{g_W} \tilde{C}'(W),\]

(37)

which suggests that the decrease in profits over the last added capacity should be equal to the associated increment in consumer surplus, i.e. to the "marginal saving" in the disutility from waiting, discounted by the shadow value of the budget constraint (for a similar result, in connection with different markets, see Bergantino et al., 2006).

7 Ranking allocations

We can write social welfare as the sum of consumer surplus and profits:

\[SW = U(X) - pX - \tilde{C}(W) + pX - \tilde{C}(V) - C(X) = U(X) - pX - \tilde{C}(W) + \Pi \]

It follows that:

\[\frac{\partial SW}{\partial p} = -X + \frac{\partial \Pi}{\partial p} \]

(38)

\[\frac{\partial SW}{\partial V} = \frac{g_V}{g_W} \tilde{C}'(W) + \frac{\partial \Pi}{\partial V} \]

(39)
**Social optimum** Substituting (16) in (38) and (17) in (39), at social optimum we have:

\[
\frac{\partial SW^{FB}}{\partial p} = 0 \quad (40)
\]
\[
\frac{\partial SW^{FB}}{\partial V} = 0 \quad (41)
\]

**Competitive market** Substituting (21) in (38) and (22) in (39), in the competitive market we have:

\[
\frac{\partial SW^{C}}{\partial p} = \frac{g_v}{g_w} \frac{\partial W}{\partial p} \tilde{C}'(V) < 0 \quad (42)
\]
\[
\frac{\partial SW^{C}}{\partial V} = \frac{g_v}{g_w} \tilde{C}'(W) + \frac{g_w}{g_v} \tilde{C}'(V) \frac{\partial W}{\partial V} > 0 \quad (43)
\]

**Monopoly** Substituting (23) in (38) and (24) in (39), in the monopoly case we have:

\[
\frac{\partial SW^{M}}{\partial p} = -X < 0 \quad (44)
\]
\[
\frac{\partial SW^{M}}{\partial V} = \frac{g_v}{g_w} \tilde{C}'(W) > 0, \quad (45)
\]

**Second best** Finally, substituting (35) in (38) and (37) in (39), in second best we have:

\[
\frac{\partial SW^{SB}}{\partial p} = -\frac{\lambda}{1+\lambda} X \quad (46)
\]
\[
\frac{dSW^{SB}}{dV} = \frac{\lambda}{1+\lambda} \frac{g_v}{g_w} \tilde{C}'(W) \quad (47)
\]

Thus, both in a competitive market and in the monopoly case, price is above social optimum and vacancies below. Assuming Social Welfare (SW) quasi-concave in V, the distortion produced by the competitive market in terms
of vacancies is larger than the one resulting in the monopoly case \((\frac{\partial SW^C}{\partial V} > \frac{\partial SW^M}{\partial V})\). Comparing the competitive market with second best is most relevant since if the latter allocation is better than the one deriving from competition, there is a rational for regulation. Given the same assumption about SW, there is less distortion at second-best than in the competitive market (in terms of the number of taxi). \textit{A priori}, the comparison is not clear in terms of prices.

## 8 Decentralisation

The result that second best allocation is strictly better that the competitive market gives a rational for regulation. Let us examine the decentralised solution when the regulator faces a profit maximising firm.

Consider a price-cap that takes account also of the quality dimension represented, in this context, by the unused capacity, V, and, indirectly, thus, by the value of waiting time. Define it as follows (De Fraja-Iozzi, 2004 and Billette de Villemeur, 2004):

\[
\alpha p - \beta V \leq \bar{p}
\]  

(48)

where \(\alpha\) and \(\beta\) are the weights attributed to the price and to the vacancies (or "unused capacity"). Equation (48), which might be referred to as a "generalised price-cap" (for its property of taking into account, although indirectly, the time component of the price of the trip), requires the firm to choose price and vacant capacity such that the difference between their weighted sum is lower than the exogenously determined level \(\bar{p}\). When \(\beta\) is equal to 0, vacancies are not regulated. If, instead, the quality dimension represented by V is relevant \((\beta > 0)\), the firm can alter its price constraint by increasing the level of unused capacity and, thus, obtain an increase in the allowed price: the unused capacity level determines the upper limit for fares or, stated in another way, tariff setting determines a minimum vacancy level. With this set up, the firm is thus free to use its knowledge of demand in order to choose the price and vacant capacity, provided that the "generalised price" does not exceed \(\bar{p}\).

The Lagrangian associated to the problem of the regulated firm is:

\[\mathcal{L} = pX - C(X) - \tilde{C}(V) + \mu [\bar{p} - \alpha p + \beta V].\]

where \(\mu\) is the Lagrangean multiplier associated with the constraint in (48).

The FOC are given by:

\[
\frac{\partial \mathcal{L}}{\partial p} = X + [p - C'(X)] g_w \frac{\partial W}{\partial p} - \mu \alpha,
\]

(49)

\[
\frac{\partial \mathcal{L}}{\partial V} = [p - C'(X)] \left(g_V + g_w \frac{\partial W}{\partial V}\right) - \tilde{C}'(V) + \mu \beta.
\]

(50)
Assume that the (exogenous) weights $\alpha$ and $\beta$ are such that:

$$\alpha = X^* \quad \text{and} \quad \beta = \frac{g_V}{g_W} \tilde{C}'(W^*)$$

where the subscript $*$ refers to the second-best allocation. (49) and (50) rewrite:

$$\left(1 - \mu \frac{X^*}{X} \right) X + [p - C'(X)] \frac{\partial X}{\partial p} = 0$$

$$\frac{\partial \Pi}{\partial V} + \mu \frac{g_V}{g_W} \tilde{C}'(W^*) = 0$$

Assume furthermore that $p$ - fixed by the regulator - is adjusted so that firm's profits goes to zero. It must be the case that

$$1 - \mu \frac{X^*}{X} = \frac{\lambda}{1 + \lambda}$$

so that $\mu = 1/(1 + \lambda)$ and

$$\frac{\lambda}{1 + \lambda} X + [p - C'(X)] \frac{\partial X}{\partial p} = 0,$$

$$\frac{\partial \Pi}{\partial V} + \frac{1}{1 + \lambda} \frac{g_V}{g_W} \tilde{C}'(W) = 0.$$ 

In order to implement the optimal solution, it is thus sufficient to compel the firm to offer services such that their "generalised price" ($\tilde{p}$) does not exceed its second best optimal values.

The regulatory mechanism just described despite its simplicity, appears to be implementable. An iterative mechanism inspired to Vogelsang and Finsinger (1979) should allow to determine the appropriate weights using only past accounting information (e.g. book-keeping data)\(^1\). In this particular case, the information needed would refer to the average value of time and to the marginal impact of an additional taxi in the fleet on the expected queuing time.

9 References


\(^1\)It has been shown, in fact, that under some well-established conditions, these coefficients will converge to their optimal values. On the convergence of this mechanism, see Etienne de Villemeur 2003. On these mechanisms and their limits, Laffond and Tirole (1993, section 2.5.2) and, more recently, Law (1997) and Cowen (1998).
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