MARGINAL COMMODITY TAX REFORMS: A SURVEY

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Marginal commodity tax reforms: a survey

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Abstract

We summarize the main features and results of the literature on marginal commodity tax reforms as initiated by Ahmad and Stern (1984) and developed further by Yitzhaki and Thirsk (1990). We establish new links with other fields of research, namely the literature on the use of equivalence scales and the one on poverty measurement. We also critically examine some issues associated with the implementation of marginal tax reforms, with special reference to the estimation of welfare weights and of revenue effects.

1 Introduction

Thirty years ago Martin Feldstein (1975) noted that the knowledge of optimal taxes may be useless for practical purposes since actual tax changes are "slow and piecemeal". Consequently, he proposed to shift the emphasis from tax design to tax reform. The latter was then defined by Guesnerie (1977) as a vector of small tax changes which are "feasible" and "satisfactory" according to the Pareto criterion. King (1983) noted that when revenue-neutrality is also assumed the distributional impact of the reform can be easily related to its efficiency gain (or cost). Building on this literature, Ahmad and Stern (1984) investigated marginal commodity tax reforms defined as vectors of small tax changes which increase welfare, as measured by a given social welfare function, without decreasing revenues. Since the beginnings of their analysis, Ahmad and Stern (1984, p. 261) state that the theory of marginal reforms is limited in scope since it indicates directions of tax changes while saying nothing about the size of the reform, and, furthermore, it leaves unanswered the question of which (of the many) directions should be followed. Nevertheless, marginal reforms have proven to be quite popular among the economic profession. Few years ago Ranjan Ray (1997) published on the Journal of Economic Surveys a comprehensive review on the theory and practice of commodity taxation in which only 1 out of 6 sections was devoted to marginal commodity tax reforms. Since then, however, the literature on marginal commodity reforms has rapidly grown and now a considerable number of papers has been published on highly

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ranked academic journals. This expansion has followed 3 main paths: i) the implementation and the extension of the original framework designed by Ahmad and Stern (1984); ii) the development by the *daltonian school* of an alternative framework starting from a less demanding structure of welfare weights (Yitzhaki and Thirsk, 1990; Yitzhaki and Slemrod, 1991; Mayshar and Yitzhaki, 1996) and, more recently iii) the interpretation of marginal commodity tax reforms as poverty-reducing policy changes (Makdissi and Wodon, 2002; Liberati, 2003).

There are two main reasons behind this academic success. First, as stressed by Ray (1997, p. 367), ”marginal commodity tax reforms calculations impose far less data requirements than optimal commodity tax rates”. This is particularly important when available databases are relatively poor as it usually happens in developing countries. Indeed, marginal tax reforms appear to be particularly suitable for this kind of countries (Ahmad and Stern, 1987 and 1991). Second, as noted by Feldstein (1975) and remarked more recently by Slesnick (1998, p. 1253) ”reforms are often restricted due, for example, to political considerations, so that large changes in tax rates are infeasible”.

Despite the large number of papers written in recent years, a comprehensive and updated review of the theory and practice marginal commodity tax reforms is still missing. The first purpose of the present paper is to fill in this gap. Our main motivation is that marginal commodity tax reforms are a policy tool whose importance will be increasing in the nearest future, in both developed and developing countries, given that direct taxation encounters growing difficulties of political and economic nature. Our second purpose is to establish some useful and interesting links with other fields of research, namely the literature on the use of equivalence scale and the one on poverty measurement. Our third purpose is to analyze more accurately than it is currently done some issues associated with the implementation of marginal tax reforms, with special reference to the estimation of welfare weights and of revenue effects.

The paper is organized as follows. Sections 2 and 3 are respectively devoted to an illustration of the two main streams of the literature: the one originated by Ahmad and Stern (1984) and the *daltonian school* whose starting point can be considered the paper by Yitzhaki and Thirsk (1990) . Section 3.1 is devoted to the analysis of some issues arising in the calculation of welfare weights. Section 4 explains how marginal commodity tax reforms are implemented when social heterogeneity is taken into account and, by doing so, establishes a link with the literature on equivalence scales. Section 5 reviews the literature on the econometric estimation of revenue effects of marginal reforms and evaluates its impact on marginal commodity taxation theories. Section 6 considers the potential of marginal indirect tax reforms as poverty-reducing policy tools, starting from the analysis by Makdissi and Wodon (2002) and evaluating its implications with a moveable poverty line. Section 7 provides some concluding remarks.
2 Marginal commodity tax reforms

Consider the problem of maximizing a social welfare function $W(V^1(q, y), \ldots, V^n)$ where $V$ is the indirect utility function, $q$ are consumer prices and $y$ is income and where, for the time being, $i = 1, \ldots, n$ are single individuals. Assume that a revenue constraint is written as $R = \sum_c \tau^c X^c$ where $R$ is total revenue raised through commodity taxation, $\tau^c$ is the specific tax rate on commodity $c$ and $X^c$ is aggregated demand. Denote marginal revenue from the taxation of $c$ as with $MR^c = \partial R/\partial \tau^c$. If one defines for a pair of commodity $(j, l)$ with $j \neq l$

$$\theta^l \equiv -\frac{dW}{d\tau^l}/MR^l; \theta^j \equiv -\frac{dW}{d\tau^j}/MR^j$$

(1)

where both $MR$'s $\neq 0$, then at the optimum it necessarily holds that

$$\theta^l = \theta^j$$

(2)

This equality is just saying that the marginal social cost of funds raised through commodity taxation (i.e the marginal loss in welfare relative to the marginal increase in revenues caused by taxation of a given commodity) should be equal across all commodities. Suppose that (2) does not hold strictly, i.e., for example, that

$$\theta^l \geq \theta^j$$

(3)

which is saying that the marginal social cost of funds raised through taxation of $l$ is not lower than the marginal social cost of funds raised through taxation of $j$. Intuitively, in this case existing marginal rates are not optimal and an increase in constrained social welfare can be obtained by increasing taxation on $j$ and decreasing taxation on $l$, keeping revenues unchanged. This is the idea of a marginal tax reform. More precisely, a **marginal tax reform** (Ahmad and Stern (1984)) is a reform such that, when (3) holds,

$$d\tau^l = -d\tau^j \frac{MR^j}{MR^l}, d\tau^j > 0$$

(4)

i.e. a marginal tax reform, while keeping total revenue unchanged, subsidizes marginally $l$ by marginally taxing $j$ when (3) holds. Such a reform actually has a nonnegative impact on welfare as it can be seen immediately by totally differentiating $W$ with respect to tax changes when (4) holds.

Two theoretical limitations, however, emerge. First, although it is ensured that a marginal tax reform does not decrease welfare, i.e. that it goes towards a ‘good’ direction, nothing guarantees that the reform will reach optimality (i.e that (2) is verified after the reform is accomplished). This implies that, even when there is a unique optimum, there might be other reforms which can be preferred to the one considered, and therefore external criteria to choose among different reforms, such as administrative convenience, political acceptability or
reliability of estimated parameters, are called for (Ahmad and Stern, 1991, pp. 61-62).

Second, only the direction is specified, while the size of the reform is not: (4) is just suggesting that taxation on \( j \) should be marginally increased and taxation on \( l \) marginally decreased so that aggregate revenues are unchanged, but there are many (if not infinite) combinations of tax changes satisfying (4).

It is usually claimed that these theoretical limitations allow a marginal tax reform to be less informationally-demanding than a reform aiming at optimal taxation. This is an important claim which rests nevertheless on some assumptions that need to be discussed. Using Roy’s identity and assuming also that \( dq = d\tau \) for all commodities it can be written that

\[
-\frac{dW}{d\tau^c}/MR^c = -\sum_i \frac{\partial W}{\partial V^i} \frac{\partial V^i}{\partial \tau^c} / MR^c = -\sum_i \frac{\partial W}{\partial V^i} \lambda^i x^{ci}/MR^c
\]

\[
\equiv (X^c/MR^c) \sum_i \frac{\partial W}{\partial V^i} \lambda^i s^{ci}, c = j, l
\]

where \( \lambda^i \) is marginal utility of income for individual \( i \) and \( x^{ci} \) is her consumption of commodity \( c \), \( X^c \) is aggregate demand of commodity \( c \) and \( s^{ci} = x^{ci}/X^c \) is the share of commodity \( c \) which is consumed by individual \( i \). The following notation is usually introduced

\[
\beta^i \equiv \frac{\partial W}{\partial V^i} \lambda^i; \alpha^c \equiv \frac{MR^c X^c}{X^c}
\]

where \( \beta^i \) is the social weight of a marginal variation in \( i \)'s income while \( \alpha^c \) is the reciprocal of the marginal efficiency cost of funds. The higher is the \( \alpha^c \), the lower is the marginal efficiency cost of raising funds through the taxation of commodity \( c \). In fact, \( \alpha \) increases when the marginal welfare gain of taxation (measured by marginal revenues) increases with respect to marginal welfare loss when the latter is limited to first-order effect and thus measured by aggregate demand before taxation (see Mayshar, 1990, pp. 266-67). Using (5) and (6), (3) is rewritten as

\[
\alpha^j \sum_i \beta^i s^{ji} \geq \alpha^l \sum_i \beta^i s^{li}
\]

so that a marginal tax reform is such that (4) and (7) are satisfied, and it ultimately depends on the distributional characteristics (Feldstein, 1972) of the two commodities and on their marginal efficiency cost of funds. Note that derivatives of aggregate demands enter in (7) since \( MR \)'s depend on own and cross price-substitution effects (we shall return on this point in section 5). An alternative presentation of (7) (see Ray, 1997, eq. (24) p. 367 and Madden (1996), eq. (3) p. 559) is expressed in terms of elasticities. If the revenue constraint is ignored, only the distributional effect is taken into account, so that \( \alpha \)'s will disappear from (7) which will then boil down to a comparison between
the distributional characteristics of the two commodities (see Newbery, 1995, eq. (3) p. 851).

It is important to recall that the use of Roy’s identity is valid as long as marginal price changes are considered. While this assumption does not restrict the theoretical scope of the reform as defined previously, it does urge the researcher to be particularly cautious when implementing the reforms (see section 3.1). The second assumption that was made above, i.e. \( dq = d\tau \), while common to all the literature on commodity taxation, really rests upon the assumption of either perfect competition or of a small open economy facing world markets (Dixit, 1975, p. 109; Cragg, 1991, p. 126).

The framework constituted by equations (1)-(7) has been expanded in various directions. Madden (1995) has amended it to take also labour into account. This implies that, in principle, leisure enters the social welfare function and that wage taxes enters the revenue constraint. If labour is not a commodity involved in the reform (i.e. wage taxes are not modified), then the \( \alpha \)'s parameter in (7) would be explicit functions of the effect on labour supply of the marginal tax changes (see eq. (11) in Madden (1995)). When wage tax rates are (marginally) changed, revenues are modified accordingly and a very similar expression to (5) is obtained for leisure (see eq. (10) in Madden, 1995). However, such an extension correspondingly requires additional information (data on hours worked and wage rates).

A second development has been to include externalities into the framework. In the most complete setting, Mayeres and Proost (2001) assume that an externality may affect both directly the utility level and the demand for different commodities (the latter is called feedback effect). When externality enters the utility level, (7) is modified since \( \beta \)'s multiply (not only demands for \( l \) and \( j \) but also) the change in the level of externality induced by the reform (see eq. (3) in Schob, 1996, p.539). Such a change depends upon the substitution/complementarity effect between the externality-generating commodities and commodities \( l \) and \( j \). On the other hand, when the feedback effect is produced, also the \( \alpha \)'s parameters in (7) are affected. More precisely, \( MR^c \) will depend also on the marginal change in \( X^c \) determined by the change in the level of externality induced by the reform (see the denominator of eq. (11), p. 347 in Mayeres and Proost, 2001).

Another theoretically feasible extension would be to drop the complete shifting assumption and to consider a forward shifting rate \( \rho^c = dq^c/d\tau^c \neq 1, c = j, l \). This would imply to divide \( \alpha^j \) and \( \alpha^l \) in (7) by the respective shifting rate. Further steps would depend on the nature of the market considered. Scade (1985) shows that, if there is imperfect competition between symmetric firms, the forward shifting rate will be higher than 1 when the elasticity of the slope of the inverse demand function for the given sector is higher than 1. In a similar vein, Stern (1987, pp. 72-73) argues that the forward shifting rate in non competitive markets depends not only on the elasticity of demand, but also on ”the elasticity of elasticity”. A large set of data should thus be available to drop the complete shifting assumption. Moreover, Stern argues that the assumption of complete forward shifting may be less implausible than what it seems at a first glance.
Turning back to (7), it can be noted that four items of information are required:

i) consumer’s demands (consumption levels);
ii) tax rates;
iii) welfare weights;
iv) aggregate direct demand derivatives or, equivalently, elasticities of these aggregate demands.

Overall, these requirements are far less demanding than those for optimal tax rates. Items i) and ii) are standard in indirect tax analysis, although they might not be easy to obtain in empirical applications. A typical difficulty, for example, is given by tax systems where there a number of implicit taxes such as import duties and excise taxes (see Ahmad and Stern, 1991, pp. 116-122, for a discussion and treatment). Items iii) and iv), however, are more central to the literature considered in this paper. For expositional convenience, we first examine the choice of welfare weights.

### 3 Welfare weights

Ahmad and Stern (1984) calculate the welfare weights by taking a utility function of the Atkinson-type, specified as (now \( i \) is a subscript)

\[
W = \sum_{i=1}^{n} k \frac{y_i^{1-\varepsilon}}{(1 - \varepsilon)}
\]

where \( \varepsilon \) is the inequality-aversion parameter (\( \varepsilon \neq 1 \)). This implies that

\[
\beta_i = ky_i^{-\varepsilon}
\]

Assuming \( k = 1 \) and imposing a normalization such that \( \beta_1 = y_1^{-\varepsilon} = 1 \) yields the following expression

\[
\beta_i = \left( \frac{y_i}{y_1} \right)^{-\varepsilon}.
\]

Once welfare weights are specified, and provided items i), ii) and iv) are all available, a welfare judgement concerning different reforms can be given. Note that if there are more than 2 commodities they can be ranked by calculating the value of \( \theta \) for each commodity and for every value of the inequality aversion parameter. We define the latter as the 'ranking commodities' procedure (not to be confused with the ranking of individuals or households that we will be discussing next) and will examine it in section 5.

A large part of the literature has followed Ahmad and Stern imposing a well specified weighting function: among the most recent examples, \( \beta_i = (y_i)^{-\varepsilon} \) has been used, although in a socially heterogeneous context (see next section), by Kaplanoglou and Newbery (2004, p. 239); Creedy (2001; p. 462) and Liberati
(2001; p. 33). Clearly, one may conceive other social welfare functions, such as, for example, the Kolm-Pollack utility function (Cragg, 1991, p. 131).

However, a different approach has been proposed by Shlomo Yitzhaki and his associates in a number of papers published in the Nineties (Yitzhaki and Thirsk, 1990; Yitzhaki, 1990; Yitzhaki and Slemrod, 1991; Mayshar and Yitzhaki, 1995; Yitzhaki and Lewis, 1996; Yitzhaki, 1997). According to Mayshar and Yitzhaki (1995, p. 793) "two schemes have been utilized to tackle the value-judgement requirement" in the literature on tax reforms:

1) the Paretian approach which "seeks to avoid the need for welfare comparisons altogether". In practice, this approach will yield no results, since reforms must benefit everyone to be approved.

2) the Ahmad and Stern’s approach based on "imposing an arbitrary structure of social welfare to enable interhousehold comparisons".

Mayshar and Yitzhaki (1995) propose an intermediate approach based on the assumption that "there exists some observable characteristic (...) by which households can be socially ranked". This is similar to the daltonian criterion in that it requires only a rank and not a measurement of social welfare. Moreover, marginal reforms should ensure that no reranking is caused. For these reasons this stream of literature is called the daltonian school in the following. To see what contribution is provided by the daltonian school, assume that the $n$ individuals are ranked according to a specified criterion. For example, when income is used one has

$$y_1 < y_2 < ... < y_n, i = 1, ... n$$  \hspace{1cm} (11)

and it can be assumed that

$$\beta_1 \geq \beta_2 \geq ... \geq \beta_n \geq 0, i = 1, ... n$$  \hspace{1cm} (12)

Now, when (12) is the only available information about weights, (7) cannot be computed. However, a new set of sufficient conditions for a marginal increase in welfare can quite easily be identified. The objective is

$$dW = \sum_{i}^{n} \beta_i dy_i \geq 0$$  \hspace{1cm} (13)

where $dy_i$ is the variation of equivalent income i.e the variation of income which is equivalent, in utility terms, to the variation of prices. From Roy’s identity and using also the revenue neutrality condition (see (4) above) it can be immediately noted that, when only two tax rates are changed, this variation is equal to

$$dy_i = - \left[ s^i - \frac{\alpha^i}{\alpha^j} s^{ij} \right] X^i d\tau^i$$  \hspace{1cm} (14)

It is useful to rewrite (13) as (Mayshar and Yitzhaki, 1995, p.797)
\[ dW = \sum_{k=1}^{n} \beta^k \left( \sum_{i=1}^{k} dy^i - \sum_{i=1}^{k-1} dy^i \right); i, k = 1, ..., n \] (15)

or equivalently as

\[ dW = \sum_{k=1}^{n} \left[ (\beta^k - \beta^{k+1}) \sum_{i=1}^{k} dy^i \right] \geq 0; i, k = 1, ..., n \] (16)

Now, combining (14) and (16), and recalling from (4) that \( d\tau^l < 0 \), it is clear that the set of \( n \) conditions

\[ \sum_{i=1}^{k} \left( s^{li} - \frac{\alpha_j}{\alpha^l} s^{ji} \right) \geq 0, k = 1, ..., n \] (17)

is sufficient to satisfy (13) (as claimed firstly by Yitzhaki and Thirsk, 1990, pp. 5-7) when \( l \) is the subsidized commodity, i.e when \( d\tau^l < 0 \), since the term \( (\beta^k - \beta^{k+1}) \) is always non-negative when assumption (12) holds.

Summing up, a *daltonian tax reform* is a reform which, when (17) holds, changes the marginal tax rates as indicated in (4). While (7), i.e the condition for a marginal tax reform, is a comparison between values of total consumption of two commodities (so it is a comparison between two numbers), (17) is a comparison between values of shares of consumption of two commodities evaluated at all possible ranks, starting from the lowest. The share of consumption of a given commodity at a given rank is the value of the concentration curve of that commodity at the given rank: it indicates how much of the good is consumed by the poorest decile, by the poorest 20% and so on until all the population is considered (see Kakwani, 1977 for a formal treatment of concentration curves). So, (17) requires dominance of the concentration curve for commodity \( l \) with respect to the concentration curve for commodity \( j \).

There are a number of feasible extensions of the daltonian school’s approach. Lundin (2001) has included aggregate externalities into the analysis, basically following the approach by Schob (1996) and thus ignoring any feedback effect as the one considered by Mayeres and Proost (2001). As already argued in previous section, this means that the externality enters the utility function and therefore the expression for \( dy^l \) in (14) should be modified by considering not only (individual and aggregate) demands but also marginal changes in utilities caused by marginal changes in externalities induced by the tax reform (see equations 2.5 and 2.6 in Lundin, 2001, p. 820).

To compare daltonian tax reforms with marginal tax reforms we may say that the two are theoretically limited to the same extent but that the former is less demanding, being based on an ordinal rather than a cardinal approach. More precisely, among the items i)-iv) listed at the end of section 2, the daltonian tax reform does not require iii), i.e the specification of a weighting function. Therefore, the ‘daltonian’ approach may be seen as less arbitrary than the one
on which marginal tax reforms were originally based. On the other hand the possibility to evaluate the welfare impact as a function of an inequality-aversion parameter may be seen as a source of flexibility rather than a source of arbitrariness. To sum up, the choice between the two approaches rests ultimately on researcher’s beliefs and priorities.

Whatever approach is chosen, the measurement of welfare weights raises also some implementation issues that will be discussed in next subsection.

3.1 Implementation issues

In his review on empirical approaches to welfare measurement, Slesnick (1998, p. 2153) notes that the initial impression about the possibility to evaluate welfare effects of marginal price changes without estimating demand functions econometrically turned out to be "illusory". There are two arguments supporting this judgement, and they are both based on a previous contribution by Banks et al (1996).

The first argument is that first order approximations can be quite inaccurate for many price changes (i.e. tax reforms) that are of practical interest. In fact, Banks et al. (1996, pp. 1234-1238) show that when a new tax of 17.5% is raised on a previously untaxed commodity (clothing) to have a uniform commodity tax system in the UK, using first order approximations would lead to a substantial bias in the estimated effect on total expenditures. Large changes of tax rates would require to use a second order approximation which would add a term to the pre-tax demand level. This is equivalent to say that with large changes of tax rates the calculation of the welfare effect cannot be simply based on the Roy’s equivalence, as we already know from our brief discussion in section 2. Indeed, Banks et al. (1996, p. 1238) acknowledge that "for small reforms (...) suitable first order approximations can work very well". However, the results obtained by Banks et al. (1996) emphasize the importance to be particularly accurate in the selection of marginal tax reforms. The rule of thumb proposed by Yitzhaki and Lewis (1996, p. 543) is to consider a reform to be marginal when it does not change equivalent incomes by more than 10 percent. From the work of Banks et al. (1996) another useful suggestion is to carry on a sensitivity analysis on the difference between the first and the second order approximation (on the basis of a Taylor’s expansion, see (equations (17) and (18) in Banks et al. (1996), p. 1233).

The second argument put forward by Slesnick (1998, p. 2153) deserves more attention. It is based on Theorem 1 by Banks et al. (1996, p. 1231) whose corollary can be expressed as follows: the welfare weights $\beta$’s are independent of prices (or, equivalently, are a function of income only) if and only if the indirect utility functions are homothetic. This means that if preferences are not homothetic one should evaluate how prices influence weights, which, since prices enter indirect utilities via marshallian demands, in turn requires an econometric estimation of demand functions. Despite Slesnick’s remark, this result has been ignored by the literature which has invariably treated welfare weights as
price-independent. However, its impact should be considered carefully for two reasons. First, homotheticity is hardly found in empirical applications. Second, an homothetic indirect utility function yields unitary expenditure elasticities for all commodities and, under such a condition, we know from Theorem 1 by Kakwani (1977) that it could be difficult to find dominance between a given pair of concentration curves. Therefore homotheticity should be excluded since it is, in general, empirically untenable and, in particular, incompatible with daltonian tax reforms. The importance of Slesnick’s remark, therefore, boils down to verify how welfare weights react to prices when utility functions are not homothetic. To provide some insights on this problem, we consider the LES specification of demand functions which is quite popular in the literature as we shall see briefly. The indirect utility function for this demand system is given by (Pollack and Wales, 1992, p. 10)

\[ V(q, y') = y' - \sum_c q_c b_c \prod_c q_c \]  

where, for every commodity \( c \) whose price is \( q_c \), \( b_c \) denotes the necessary bundle for that commodity while \( \varphi_c \) is the proportion of remaining income which is used to buy quantities of \( c \) exceeding the necessary bundle, where remaining income is income less the expenditure on all necessary bundles. The marginal utility of income is thus equal to

\[ \lambda' = \frac{1}{\prod_c q_c} \]  

which clearly is a function of prices as claimed by Banks et al (1996). Let us now consider how \( \beta \)'s will change when (19) holds and the tax rate on two commodities is changed, while prices on other commodities and \( dW/dV \)'s are held constant. Before the tax reform the denominator of (19) is given by \( \text{den} = K q_l q_j \) where \( K = \prod_t q_t, t \neq l, j \). Totally differentiating \( \text{den} \) with respect to \( q_l \) and \( q_j \), using (4) and the assumption \( dq_c = d\tau_c, c = j, l \) yields

\[ \Delta \text{den} = \varphi_j K q_l^{\varphi_j} q_j^{\varphi_j-1} dq_j + \varphi_l K q_j^{\varphi_l} q_l^{\varphi_l-1} dq_l = \text{den} \left( \frac{\varphi_j}{q_j} - \frac{MR_j}{MR_l q_l} \right) dq_j \]  

For a given size of the reform, i.e for a given value of \( dq_i \), the difference in marginal utility of income depends on the absolute value of the difference within round brackets. For example, for given initial prices and given marginal revenues, the latter difference increases as the ratio \( \varphi_j/\varphi_l \) diverges from unity. This is saying, for instance, that the absolute difference in welfare weights may be important when the subsidized commodity is a necessity and/or the taxed one is a luxury. On the other hand, ceteris paribus, the difference in marginal utility of income decreases as \( dq_i \) decreases. The latter remark is saying that the smaller (the more marginal) is the reform, the smaller is the bias introduced by ignoring the change in welfare prices, when a LES specification of the demand system is adopted. Thus, a LES is a case in which the claim by Ahmad and
Stern (1984, p. 279) that welfare weights can be treated as convenient "local approximations" even if they are not strictly valid for all prices and all incomes seems justified. However, a different outcome may arise when non linear demand systems are adopted. Moreover, this approach overlooks the difficulty of choosing the 'right' demand system, which may depend, also, on price sensitivity of different specifications (see Ray, 1999, p. 702).

4 Heterogeneity

The interpretation either of a marginal (formulae (4) and (7)) or of a daltonian (formulae (4) and (17)) tax reform must take into account the fact that consumption units in the real world are heterogeneous households rather than individuals. We discuss here how the issue of heterogeneity is dealt with first by papers implementing the marginal tax reform approach and secondly by the daltonian school.

Equivalence scales are used by the majority of researchers. While Ahmad and Stern (1984; 1991) used simply per capita income, expenditure per equivalent adult is chosen by Madden (1995) and by Liberati (2001) while real consumption per equivalent adult is adopted by Newbery (1995) and by Kaplanoglou and Newbery (2004). Cragg (1991, p. 139) uses a peculiar method to equivalize data when households are aggregated in income classes.

While the use of equivalence scale is widespread, a complete framework for the case of heterogeneous households is still missing in the existing literature. We now briefly sketch it by taking the structure of section 2 as our starting point. Suppose that $i$ is interpreted as a generic household behaving, as a whole, as an individual, so that the Roy’s equivalence is still valid at the household level. To write the formula corresponding to (5) for the heterogeneous case a social welfare function must be defined. Ebert (1997, Proposition 1, p. 237) shows that, if additivity of $W$ is assumed, there is a precise correspondence between the equivalence scale that it is used and the social welfare function that it is implied. This means that for every equivalence scale, an additive social welfare function yielding the same welfare judgement can be identified. In particular, when equivalent expenditure per equivalent adult is used, the latter result is generalized as follows. Assume that each household $i$ belongs to a group $g$, $g = 1, ..., G$ defined according to the social features of the households (size, presence of old or disabled people, and so on). Then choosing equivalent expenditure per equivalent adult corresponds to (i.e yields the same welfare judgement as) the following social welfare function (Ebert, 1997, p. 243)

$$W = \sum_{g=1}^{G} \sum_{i \in g} \delta^g V(q, y^i / \delta^g)$$

where the monetary income $y_i$ is replaced by $\delta^g$ equivalent adults having
each equivalent income $y_i/\delta^g$ and where $\delta^g = 1$ for the reference group (usually
the single) whose utility is $V(\cdot)$. This implies that

$$\beta^{i\epsilon g} = \frac{dW}{dy_i} = \delta^g \frac{dV^{i\epsilon g}}{dy^i}$$

(22)

If one assumes that

$$V^{i\epsilon g} = \frac{(y^i/\delta^g)^{1-\varepsilon}}{(1-\varepsilon)}$$

(23)

and interpret $\varepsilon > 0$ as reflecting risk aversion at the individual level, the
weight immediately rewrites as

$$\beta^{i\epsilon g} = (y^i/\delta^g)^{-\varepsilon}$$

(24)

This reasoning provides a rationale for using (24) when implementing mar-
ginal or daltonian tax reforms to socially heterogeneous context, as it is often
done in the literature (see for example Liberati, 2001, p. 33).

Turning now to the daltonian school it is interesting to note that, despite this
stream of literature originates from a refusal to endorse an arbitrary weighting
function, equivalence scales, which are arbitrary by definition, are often used in
is easy to see that by allowing for equivalence scales the daltonian approach in
the heterogeneous context yields a set of inequalities very similar to (17) . To see
why, it suffices to note that (11) is replaced by a ranking based on $y_i/\delta^g, i \in g$.

Adopting equivalence scales is very convenient in empirical terms. Setting
aside econometric issues (see next section), and provided that $\alpha^j \geq \alpha^l$, a neces-
sary condition for (17) when equivalent expenditure is used is that the concen-
tration coefficient for commodity $l$ is not higher than the concentration coe-
efficient for commodity $j$. From the definition of a concentration coefficient (see
Lambert, 2001, p. 33) this provides a practical criterion to search for good
candidates for a reform by using

$$\frac{\text{cov}[x^{iL}, F(y^i/\delta^g)]}{X^i} \leq \frac{\text{cov}[x^{ij}, F(y^j/\delta^g)]}{X^j}, i \in g$$

(25)

where $F$ is the distribution of equivalent expenditure. Clearly, (25) applies
also to the homogeneous case by simply setting $\delta^g = 1\forall g$. Inequality (25) can
have a number of different and interesting interpretations (see Yitzhaki, 1997,
pp. 247-248 ) but it is only a necessary and not a sufficient condition for (17) in
both versions (homogeneous and heterogeneous). In other words, if a pair of
commodities $(j, l)$ satisfies (25) they are plausible candidates for a reform, but
dominance should still be verified.

However using equivalence scales to implement daltonian tax reforms may
be somewhat inconsistent with the philosophy inspiring the daltonian approach.
If one does not like the idea to attribute welfare weights to single consumption
units one might also dislike the idea that a money metric measure of well being can be used to compare welfare of households having different features. The alternative is to apply to this context the sequential dominance criterion, as done by Mayshar and Yitzhaki (1996, pp. 405-407) who also provide a formal proof. We content here with the intuition behind their reasoning. If the $G$ groups of households are formed according to a variable measuring needs, say $n$ (for example, the size of the household) the household rank is a function not only of income but also of $n$, so that $y$ and $n$ together define the living standard of every household. This in turn implies that the total variation of welfare is modified. More precisely, (15) should be rewritten by subtracting from the variation in equivalent income of all households whose pre-tax living standard is not higher than $k$’s the variation of all households whose pre-tax standard living is surely lower than $k$’s. These comprise not only poorer households, but also households having the same income but a higher $n$ (see equation (8) in Mayshar and Yitzhaki, 1996, p. 405). Having done that, (15) can be rewritten by reaggregation (see equation (9) in Mayshar and Yitzhaki, 1996). Intuitively, rather than comparing $\beta^k$ only with the weight of the richer individual, i.e with $\beta^{k+1}$, as in (15), one will have to compare it also with the weight of households having the same income but a lower $n$ and with those having both a higher income and a lower level of needs. It turns out that the assumption which parallels $\beta^k > \beta^{k+1}$ is the following one (see the last term in equation (9) in Mayshar and Yitzhaki, 1996)

$$\beta(y_k; n_k) - \beta(y_k; n_{k-1}) > \beta(y_{k+1}; n_k) - \beta(y_{k+1}; n_{k-1})$$  \hspace{1cm} (26)

where $\beta(y_k; n_k)$ is the weight of a household having an income $y_k$ and a level of needs $n_k$, $\beta(y_k; n_{k-1})$ is the weight of a household having an income equal to $k$’s but a lower level of needs, $\beta(y_{k+1}; n_k)$ is the weight of a household having the same level of needs as $k$’s but a higher income and $\beta(y_{k+1}; n_{k-1})$ is the weight of the household having both a higher income and lower needs. Inequality (26) reflects the idea that the society become less concerned about differences in needs at higher incomes.

In practice, if (26) is valid, using the sequential approach in the implementation of (17) with heterogeneous households would involve the following steps: 1) form $G$ groups so that the level of needs is homogeneous among all households belonging to the same group 2) consider households belonging to the neediest group and rank them according to income, i.e. from the poorest to the richest 3) verify whether (17) is respected: if not the reform has an adverse distributional impact among the neediest, if (17) is verified one can move to 4) consider together households belonging to the neediest and to the next-to-neediest group, rank them according to income and verify dominance and so on until dominance is verified and all households are considered. The advantage of this approach lies not only in the fact that no arbitrary equivalence scale is required, but also that if the dominance criterion fails it can be observed where this happens. Nevertheless it is rarely applied in the implementation of daltonian reforms (exceptions are Mayshar and Yitzhaki, 1996, but only to illustrate their theoretical
results, and Liberati, 2001, pp. 40-43) as it happens also in other contexts.

In his already mentioned implementation to the case of externality-generating commodities, Lundin (2001, pp. 821-22) has extended sequential dominance to three criteria to take into account the need to consume the externality-generating commodity as an additional dimension (namely, the need to use the car producing carbon dioxide).

5 Econometric issues

We now turn to consideration of item iv) in the list of section 2. As noted by Ray (1997, p. 367) item iv) implies that we require information on demand levels and elasticities (or derivatives) only at observed point of behaviour and the (uncompensated) elasticities are aggregate rather than individual.

This means that we need less information than to calculate optimal tax rates, but the problem of the estimation of own and cross price elasticities is still present as it is evident in the definition of $MR$’s as

$$ MR^c = X^c + \sum_h \tau_h \frac{\partial X^c}{\partial \tau_h}, c = l, j \tag{27} $$

where $h$ is a generic commodity and $h$ may be $c$. Note that $MR$’s enter in (4), in (7) and in (17) and therefore both marginal and daltonian tax reforms depend on own and cross price elasticities.

In their paper Ahmad and Stern (1984, in particular pp. 277-78) use estimates of aggregate demand elasticities coming from previous econometric studies where a LES specification had been used. A linear expenditure system is used also by Creedy (2001, pp. 482-85) and, in various augmented versions, by Madden (1995, pp. 492-496). Other authors used semi-flexible functional forms, such as a normalized quadratic expenditure system (Cragg, 1991, p. 139), or flexible ones as the AIDS (Schob, 1996, p. 548).

The issue of the specification of the demand function arises naturally at this point. A number of papers (Decoster and Schokkaert, 1990; Madden 1995; Madden 1996; Ray 1999) argue that choosing the ‘right’ functional form may be less relevant when implementing (truly) marginal tax reforms following Ahmad and Stern’s (1984) approach. In particular, Decoster and Schokkaert (1990, Table 4, p. 288) provide evidence that indeed single own and cross price elasticities do differ across different demand systems. However, the outcome of the ‘ranking commodities’ procedure (see section 2 above) is not greatly modified, since the Spearman’s rank correlation coefficient between rankings of $\theta$’s derived from different specifications is reasonably high. Such a feature is somehow lost only when symmetry (of the derivative of the compensated demand) is imposed, but this theoretical restriction on single demands is questionable when only aggregate effects matters, as in (27) (for a discussion on this point, see Decoster and Schokkaert (1990, p. 293)).

These results are confirmed by Madden (1996) and to some extent by Ray (1999). However, Ray (1999, Table 4 p. 700) qualifies them in two ways:
i) correlation is particularly high when a high value of the inequality aversion parameter $\varepsilon$ is selected; ii) correlation is low across systems embodying radically different assumptions on price sensitivity of the budget shares.

High correlation between rankings of commodities obtained using different demand specifications is a valuable result for the empirical application of marginal tax reforms. The original intuition by Ahmad and Stern (1984, p. 291) that, comparatively to the implementation of an optimal tax system, "the choice of the specification of the demand system is less important (although still important) for the analysis of the reform" has thus been confirmed. However, things are less comfortable when daltonian reforms are considered, since these reforms require dominance of concentration curves which is stronger than simple 'commodity ranking'. To be sure, this is the other side of the coin: daltonian tax reforms are less demanding in terms of welfare weights, but more restrictive in their dominance criterion. Now, it is the absolute value of $MR$ which enters the dominance condition expressed by (17) (and its counterpart when heterogeneity is accounted for) and this value is directly influenced by different estimates of own and cross price elasticities. For example, taking the data provided by Decoster and Schokkaert (1990, Table 5, p. 290) one can see that a reform marginally reducing the taxation of transport while marginally increasing that on durable goods would be approved using all the 6 demand specifications considered there. On the contrary, the actual value of $MR$'s (which corresponds to the $MC$'s of Table 5 since they are calculated setting $\varepsilon = 0$) considerably varies across specifications. Nevertheless, the daltonian school usually adopts demand systems without worrying too much about specification issues: examples are provided by Mayshar and Yitzhaki (1995 and 1996) and by Yitzhaki and Lewis (1996). In some cases (Yitzhaki and Slemrod, 1991; Yitzhaki 1997) the daltonian school has assumed simply that $\alpha_j = \alpha_l$ so that revenue considerations were completely set aside. One may justify the latter when the commodities considered are price inelastic and do not have neither substitutes nor complements. In this case $MR = X$ for all commodities and $\alpha_l$'s would be equal to 1. This assumption appears to be justified only when necessities (food, some public services) are considered. However it must be acknowledged that very simple demand specifications sometimes yield results similar to more sophisticated ones (see the diagonal specification in Decoster and Schokkaert, 1990).

So far we have been discussing only about the deterministic specification of the demand function, which has been a matter of particular concern in the literature. However, also the issues of dynamic and demographic specification deserve attention. Dynamic specification may be important since consumption patterns often display time trends and dependence on lagged values. To evaluate this issue in the context of a marginal commodity tax reform, Madden (1996, Table 2, p. 563) examines how the Spearman rank correlation between $\theta_j$'s changes when, given a demand system (namely, the AIDS), its dynamic specification is changed by inserting lags and time trends. The result is summarized by saying that "the choice of a static or a dynamic demand system does matter" and therefore there is a need "for policy-makers to check the sensitivity of tax reform recommendations" to the dynamic specification of the model. Ray
(1999) considers more carefully the issue of demographic specification. In his general demand system (see Ray, 1999, equations (18) and (19) p. 695), the expenditure on a given commodity depends not only on the equivalent expenditure but also on the total number of children in the family, the number of children in each age-category and the age-category itself. Of these, no one can be excluded on purely statistical grounds, although Ray (1999) uses a nested version of the original demand system where the age effects are ruled out. This points at a consideration of social heterogeneity which is much richer than the one embodied in (25) and (26) since also α’s are seemingly affected.

It is remarkable that, despite these results on the importance of the dynamic and demographic specification no other applied work has considered these issues adequately.

6 Poverty

The most recent developments in the literature on marginal commodity tax reforms concern their interpretation as poverty-reducing reforms. Although somehow anticipated by Besley and Kanbur (1988), this interpretation has been analyzed by Makdissi and Wodon (2002) and further by Liberati (2003). Here we summarize Makdissi and Wodon’s results, establish a link with the literature exposed in previous sections and discuss possible future developments of this line of research. The starting point is the following aggregate poverty measure

\[ P = \int_0^z p(y_E(q,y), z) f(y) dy \] (28)

where \( z \) is the poverty line, \( y_E \) is equivalent income \( y \) is direct income distributed according to \( F(Y) \) whose density is \( f(y) \). Makdissi and Wodon (2002) assume, following Besley and Kanbur (1988) that the vector of reference prices used for computing equivalent income is the vector of prices before the reform. This implies that differentiating with respect to \( y \) and to \( y_E \) is equivalent (see Besley and Kanbur, 1988, p. 708). Under these assumptions it can be shown that when commodity \( j \) is taxed and commodity \( l \) is subsidized so that aggregate revenue is unchanged, the necessary and sufficient condition for a decrease in poverty as measured by \( P \) is written as

\[-X^l d\tau^l \int_0^z \frac{\partial p}{\partial y_E} \left[ s^l(y) - \alpha^l \frac{\alpha^j}{\alpha^l} s^j(y) \right] f(y) dy < 0 \] (29)

where \( s^c(y), c = j, l \) is the share of total consumption by an individual having income \( y \). In its essence, (29), which is equivalent to (6) in Makdissi and Wodon (2002, p. 230), is obtained by totally differentiating \( P \) with respect to \( y_E \) exploiting the fact that \( y_E = y \) and using (14) in the continuous framework.

Makdissi and Wodon (2002) obtain their results for a measure \( p \) defined according to some general properties. However, for the sake of illustrational purposes, let us assume that \( p \) is defined as in the FGT measure which has all of these properties. This means that
\[ p = \left[ \frac{z - y_E}{z} \right]^\gamma, \quad \gamma \geq 0 \]  

(30)

so that (29) is rewritten as

\[ \sum_{l=1}^{X_l} \tau_l \cdot \frac{z}{\gamma} - \frac{y}{E} = \sum_{l=1}^{X_l} \frac{z}{\gamma} - 1 \cdot s_l(y) - \alpha \sum_{j=1}^{s_j(y)} f(y)dy < 0 \]  

(31)

The latter expression could be derived also from equation (26) in Besley and Kanbur (1988, p. 708) imposing \( \alpha^l = \alpha^j \). Since we assumed \( d\tau_l < 0 \), (31) boils down to verify that

\[ \int_0^z \left[ \frac{z - y_E}{z} \right]^{\gamma-1} \left[ s_l(y) - \frac{\alpha^l}{\alpha^j} s^j(y) \right] f(y)dy > 0 \]  

(32)

Now consider two main possibilities: \( \gamma = 1 \) and \( \gamma = 2 \). In the first case, the FGT measure indicates the poverty gap, i.e. the amount of money necessary to eradicate poverty. In Sen’s terminology it is a crude measure since it is not sensitive to the distribution of income within the poor. If \( \gamma = 2 \) the FGT measure is distribution-sensitive so that, for example, poverty would diminish for any progressive transfer below the poverty line. If \( \gamma = 1 \), (32) is immediately rewritten as

\[ \int_0^z \left[ s_l(y) - \frac{\alpha^l}{\alpha^j} s^j(y) \right] f(y)dy > 0 \]  

(33)

If \( \gamma = 2 \), Makdissi and Wodon (2002) use the fact that \( p = 0 \) when \( y_E = z \) to integrate by parts in (32) and rewrite it as

\[ \int_0^z \left[ s_l(y) - \frac{\alpha^l}{\alpha^j} s^j(y) \right] f(y)dy > 0 \]  

(34)

The latter is the new condition introduced by Makdissi and Wodon (2002) for a marginal reform to decrease poverty according to the distribution-sensitive version of the FGT measure. Makdissi and Wodon (2002) define (33) and (34), respectively, as second and third order consumption dominance curves. They show that (33) and (34) are obtained also when \( p \) is the generalization of (30) and that similar results hold for higher orders. However, orders 2 and 3, i.e. (33) and (34) are easier to interpret and to apply.

Here we want to briefly analyze the implications of (33) and (34) when a moveable poverty line is considered, an assumption which is consistent with all the modern analysis on poverty measurement. Suppose one can identify a plausible maximum threshold \( \overline{z} \) so that \( z \in (0, \overline{z}) \). This implies that both (33) and (34) must be verified for all the values of \( z \). It is instructive to note that, if we denote with \( n(\overline{z}) \leq n \) is the rank corresponding to \( \overline{z} \) and if we ignore frequencies to simplify the notation, (33) is equivalent to
\[
\sum_{i=1}^{k} \left( s^{j,i} - \frac{\alpha^i}{\alpha^j} s^{j,i} \right) \geq 0, \; k = 1, ..., m(\overline{z})
\] (35)

which is just (17) where \( m(\overline{z}) \) replaces \( n \). This remark justifies the claim by Yitzhaki and Lewis (1996, p. 544-5) according to which sufficient conditions for a welfare-improving reform are also sufficient for a social planner "who wants to reduce the poverty gap and does not know who is poor and who is not". On the other hand, it can be shown that (34) is equivalent (again ignoring frequencies) to

\[
\sum_{i=1}^{m(\overline{z})} \theta^i \left( s^{j,i} - \frac{\alpha^i}{\alpha^j} s^{j,i} \right) \geq 0; \; k = 1, ..., m(\overline{z}) \leq n; \theta^i \equiv (k - i + 1)
\] (36)

There is a clear relationship between (35) and (36). First, (35) is a sufficient condition for (36), which can be proven by considering that \( \theta^i \) is declining in \( i \). On the other hand, in general (35) is not a necessary condition for (36), which means that (36) may hold also when (35) does not. This is evident from the two expressions, and it actually emerges also from the empirical illustration of their method proposed by Makdissi and Wodon (2002, pp. 230-231). They compare the expenditure on public transport and on medicine in Bolivia, and find that while (36) holds for all the values of the poverty lines that they consider, (35) does not above a given value of \( z \). This means that, by a poverty line higher than this given value, a marginal reform subsidizing transport by taxing medicines would not decrease the aggregate poverty gap in the equivalent income space. However, the very same reform would decrease poverty as measured by a distribution-sensitive poverty measure, as shown by the fact that (36) is satisfied.

This analysis suggests two things. First, using a poverty measure in the equivalent income space and adopting a variable poverty line whose minimum value is 0, a tax reform which decreases the poverty gap decreases poverty also when the latter is defined according to a distribution-sensitive poverty measure. The second outcome of Makdissi and Wodon’s (2002) results is that, keeping the same assumptions, decreasing poverty according to a distribution-sensitive poverty measure does not ensure that the aggregate poverty gap is decreased when these evaluations are made in the space of equivalent income and adopting a variable poverty line.

These results, which may seem counterintuitive at first, depends entirely from the fact that a variable poverty line whose minimum value is 0 is assumed, so that decreasing the poverty gap ensures that the living conditions of any "aggregation of poor" are improved. An interesting development of this line of research may be provided by a different approach where Lorenz-dominance of post-tax distribution of equivalent income with respect to pre-tax distribution of income is required in place of a distribution-sensitive poverty measure. Intuitively, and taking into account Kawani’s (1977) results, this approach leads to new sufficient conditions expressed in terms of income elasticities of different commodities.
7 Concluding remarks

The theory on marginal commodity tax reforms, initiated by Ahmad and Stern (1984), has been explored thoroughly and expanded in many directions during the last twenty years. Its advantages are to be simple and intuitive, to be politically viable, being based on small changes of tax rates, to be flexible enough to allow for many extensions and, last but not least, to be implementable at reasonable conditions. The latter feature is the logical counterpart of the limited theoretical scope of a marginal reform, which indicates only a direction for welfare improvement without ensuring that it is preferable to other possible directions and without specifying the size of tax rate changes.

The clearest indication arising from the present survey is that, while the theory seems sound and well rooted in both its versions, i.e. the Ahmad and Stern’s (1984) approach and the daltonian school (Yitzhaki and Thirsk, 1990), there are still a number of implementation issues to be solved.

First, it is now clear that the marginal character of the reform is of a crucial importance. A non marginal reform requires the estimation of second-order effects thereby going beyond the Roy’s equivalence which lies at the heart of the theory. Moreover, a non marginal reform is likely to make the ”local approximation” of welfare weights less acceptable so that price-independence of these weights would become an untenable assumption. While Ahmad and Stern (1984 and 1987) have shown to be well conscious of the importance to distinguish between marginal and non marginal reforms, the majority of the literature is somewhat less transparent in its use of the marginal reform theory. Since it does not exist a clear cut between marginal and non marginal reforms, a useful step forward would be made by stating clearly every time what is the size of the reform which is analyzed using the marginal reform theory.

Second, a statement about revenue effects is also required. One may even find it acceptable that revenue effects are completely set aside, as it sometimes happen in the literature inspired by the daltonian school, so that the ratio between α’s is made equal to 1. If such a (radical) choice is not endorsed, however, the issue of the estimation of local cross and own price elasticities is unavoidable. The latter issue has been treated mainly following the demand estimation approach. The well known result (Decoster and Shokkaert, 1990) that different specifications lead to highly correlated rankings of commodities in terms of values of their θ’s, so that the commodities which are candidates to be ‘taxed’ or ‘subsidized’ by the reform do not vary greatly across different specifications, is limited in two ways. First, it does not ensure that absolute magnitude of cross and own price elasticities do not vary across different specifications, so that dominance conditions such as those required by the daltonian school will probably change across different specifications. Second, the observed high correlation concerns only the choice of the functional form (the deterministic specification as defined by Madden (1996)) but does not involve directly the dynamic and the demographic specification of the demand system (Madden, 1996 ; Ray, 1999). Therefore, when the marginal reform theory is applied it might (depending clearly on available data) be necessary to specify the dynamic and demographic
structure of the demand system. The intrinsic difficulties in choosing the right demand system may suggest to apply non parametric estimation techniques to the estimation of price elasticities.

A final remark concerns the recent literature about poverty-reducing commodity tax reforms. We saw above that the approach by Makdissi and Wodon (2002) yields precise relationships between a reform which reduces poverty according to a crude measure such as aggregated poverty gap and a reform which decreases poverty according to a distribution-sensitive poverty measure. However, the use of a (although very general) poverty measure may seem too restrictive and one may prefer to turn to a dominance approach, i.e. an approach in which Lorenz-dominance of post-tax equivalent incomes is required, together with the reduction of aggregate poverty gap. Intuitively, this approach could be built on Kakwani’s (1977) Theorem 1 and yield results expressed in terms of income elasticities. Again, this line of research boils down to either correctly specify the demand system or to turn to non parametric estimation techniques of (price and income) elasticities.

References


