INCOME INEQUALITY AND PUBLIC GOODS PROVISION

MICHELE GIUSEPPE GIURANNO
INCOME INEQUALITY AND PUBLIC GOODS PROVISION

Michele Giuseppe Giuranno*
University of Essex
mggiur@essex.ac.uk

Abstract

This paper focuses on the question of how income inequality among regions impacts upon government decision making with regards to the provision of public goods. We model policy choices as the outcome of negotiations in a legislature compounded by regional representatives. We obtain several results. In particular, we show that greater income inequalities among regions leads to a greater degree of inefficiency in public goods provision. However, positive externalities have the effect of moderating these inefficiencies.

*I am grateful to Abhinay Muthoo for his comments on all the versions of the paper. I wish to thank Isidoro Mazza, Clive Fraser, Suresh Mutuswami, Ernesto Longobardi, Emilio Giardina, Rosa Capolupo and seminar participants at Essex, Catania, Bari, Leicester for their helpful comments. All mistakes remain my responsibility.
1 INTRODUCTION

Income disparities among regions, such as between the North and the South of Italy, create conflicts in the provision of national public goods due to the trade-off between equity and efficiency. Usually, governments introduce some redistributive mechanisms in the financing of public goods in order to reach equity targets. Tanzi (2000) argued that ”one of the major functions of a national government is precisely to redistribute income from richer regions and individuals to poorer regions and individuals through the broadly uniform provision of public goods and services”. However, the cost of greater equity is a loss in efficiency.

This paper studies the effects of regional income inequalities on government’s policy choices. In a context in which policy is negotiated by regional representatives (and not decided unilaterally by a paternalist central planner), we show that greater income disparities among regions lead to greater inefficiencies in the provision of public goods. A divergent income trend makes interregional redistributive conflicts more dramatic and it may lead to an under-provision of public goods. Our analysis suggests that conflicts are mitigated not only in the case of a converging income trend, but also in the presence of positive externalities associated with public expenditure. For example, investments in pollution control have positive externalities in terms of both public health and providing incentives for sectorial investments, which may reduce unemployment. A further interesting result captured by the model is the Giffen effect, which may occur when public goods benefits are particularly elastic. In this rare case, an increase in the public good price leads to an increase of government expenditure for that good.

Furthermore, the model centres on the provision of a national public good such as defence or pollution control. However, our key parameter may also be interpreted as the general level of government spending yearly approved in the new financial act. In this case, we assume that a disagreement would imply no government spending. The hypothesis is realistic in a country like Italy, for example, where the non approval of the new financial act has the dramatic consequence that the State is no longer able to provide any good or service.

Our model can be seen as extending Besley and Coate’s (2003) political economy analysis. They focus on the issue of which level of government should be responsible for particular taxing and spending decisions. We develop the workings of the central government focusing on the decision making process. In a model with two regions and two representatives, Besley and Coate approach decision making in the central government considering two scenarios, called the non-cooperative and the cooperative legislature. In the first, power is randomly allocated to
one of the regional delegates, who have a probability of 0.5 to choose policy by maximizing their own welfare. To some extent, we consider this case as the solution of the non-benevolent dictator. In the cooperative case, the legislature is assumed to maximise delegates’ joint surplus. We refer to this case as the social optimum or the first best policy outcome.

The main difference between this paper and Besley and Coate’s model is that we explicitly explain how regional representatives bargain over the provision of a national public good. Furthermore, public good cost is not split equally between regions, but it is covered through a proportional income tax. We highlight inter-regional redistributive conflicts, which cause a greater inefficiency in the public goods provision the larger the income gap between regions.

Lockwood (2002) also focuses on Oates (1972) and Besley’s and Coate’s (2003) issue regarding the choice between centralisation and decentralisation of fiscal policy in a political economy setting. He assumes that a central government forms policy in a legislature comprising of elected representatives from each region. Decision on a local and discrete public goods are taken by majority voting. Precisely, delegates first propose their alternative projects, and then, all alternatives are voted on according to an amendment agenda. Following Ferejohn, Fiona, McKelvey (1997), Lockwood assumes that ”the last vote pits the bill as amended against the status quo”.

Cerniglia (2003) integrates the distributive politics literature with the political economy literature of countries unions or federations. She models a legislative bargaining model by specifying the behaviour of a central legislature composed by an odd number of representatives, who are elected by regions and whose preferences differ over local public goods. Representatives make a decision by majority voting on how to allocate the amount of local public goods financed by a linear income tax or by a regional income tax. She investigated whether the credible threat of secession by any region modifies the agenda-setter proposal and hence the outcome of the legislative bargaining game. The result is that the bargaining outcome depends on both the particular representative randomly chosen to be the agenda setter and the particular voting structure of the game.

Lucas (2002) gives a theoretical approach to the transfers sharing by negotiation between central government and regions. He presents a model in which the central government, which takes action as a Stackelberg leader, first chooses the way to negotiate the transfers with regions (bilaterally or multilaterally). In the second stage, the bargaining process takes place and the federal government implements transfers to the regions. In this framework, Lucas analyses how spillovers affects the
choice of the bargaining process.

Fausto (2003) stresses the consequences caused by the disparities between wealthy and poorer regions in Italy. He argues that the fundamental means used to make a surreptitious division of the country is the financing of regions on the base of local tax revenues and of local revenues of national taxes. Inevitably, this leads rich regions to have greater financing and higher provision of public service thanks to their greater revenues. Furthermore, the underlining of the redistributive flows among regions contributes to increase regional conflicts creating a contrary atmosphere to the national cohesion.

2 THE FRAMEWORK

2.1 THE SET-UP

Consider a state comprised of two regions. Each region elects its own representative, who is identified on the median voter. The regional representatives form the legislature, which has to decide the level of a national public good (for example, defence or pollution control).

The two median voters, \( i \) and \( j \), have a different income, and different preferences on the public good. In other words, regions are not homogeneous. There are two types of goods, a public good \( g \) and a private good \( y \), which we consider to be the median voter’s income that is used for private consumption. We indicate with \( c \) the private consumption that is equal to the private income minus the public good cost. The parameter \( y \) represents the initial endowment of the median voters and it contributes to finance the public good. Instead, the parameter \( \lambda \) tells us how much a median voter prefers \( g \) with respect to \( y \). The utility function of each median voter is the sum of private and public consumption, as follows:

\[
 u_i = c_i + \lambda_i H(g) \quad \text{with } i = 1, 2
\]

where, the public good benefit function \( H(g) \), is increasing, smooth concave and satisfies the endpoint Inada condition.

The cost of the public good is \( pg \), with price \( p \) exogenous. Private consumption is equal to the private income minus the resources used to finance the public good. The legislature finances the public good by levying a proportional income tax \( t \). Once the legislature decides the quantity of \( g \), the tax rate is automatically determined as follows:

\[
 pg = t(y_1 + y_2)
\]

that gives the following tax rate \(^1\) \( t = \frac{pg}{y_1 + y_2} \). In particular, the cost paid

\(^1\)The model can be extended in order to consider the case of not uniform regional
by median voter $i$ is $ty_i = \frac{y_i}{y_i + y_j}pg = \gamma_i pg$. In other words, regions share the cost according to their relative income, indicated with $\gamma$, such that:

$$\gamma_i = \frac{y_i}{y_i + y_j}$$

This means that an increase in the income of one median voter leads to an increase of his relative cost. Furthermore, this mechanism of cost sharing implies a tax income redistribution from the richest median voter to the poorest one. The public good provision in the poorest region is partially financed by the richest region. We study how the redistributive implications of a proportional income tax influence government policy.

Median voters’ utility function becomes:

$$u_i = y_i - \gamma_i pg + \lambda_i H(g) \quad \text{with } i = 1, 2$$

### 2.1.1 DICTATOR SOLUTION AND FIRST BEST

Policy is chosen by bargaining. Before to study the bargaining outcome of this model, we first describe briefly two benchmark cases: the dictator solution and the social optimum.

First, we determine how a non-benevolent dictator chooses policy. If, for example, median voter $i$ possesses absolute executive power he is in a position to choose the level of public good which maximizes his private welfare. It is easy to establish the following result: The level of the public good $g_i$ that the non-benevolent dictator would choose is the unique solution to the following equation:

$$\gamma_i p = \lambda_i H'(g_i^D) \quad \text{with } i = 1, 2$$

Solution 1 is shown in figure 1. It states that the non-benevolent dictator would choose $g_i$ such that his private marginal cost is equal to his private marginal benefit. The private marginal cost $\gamma_i p$ corresponds to the share of the price the dictator pays for an additional unit of the public good. Instead, $\lambda_i H'(g_i^D)$ is the private marginal benefit.

The dictator always reduces the provision of the public good when its private marginal cost increases; that is: $\partial g_i^P / \partial p < 0$, $\partial g_i^P / \partial y_i < 0$, $\partial g_j^P / \partial y_j > 0$. The private marginal cost increases when there is a rise in the public good price, the dictator’s relative cost and income, or a combination of all or some of these situations. The effect of changes in the incomes of the dictator $i$ and the other median voter $j$ leads to the conclusion that the non-benevolent dictator is a free-rider. He increases the provision of $g$ when the income of the other median voter

---

tax income rate as follows: $pg = t(y_1 + y_2)$. In this case, regional representatives bargain over $g$, $t_1$ and $t_2$. 5
Figure 1: The choice of the non-benevolent dictator.
increases because this reduces his relative and marginal cost at the expense of the median voter of the other region. For the same reason he reduces the provision of $g$ when his private income increases. Instead, he increases the public good provision when his preference of the public good increases, while the preferences of the other median voter $j$ do not affect the choice of the non-benevolent dictator $i$; that is: $\partial g^D_i / \partial \lambda_i > 0$, $\partial g^D_i / \partial \lambda_j = 0$.

Now, we analyse the efficient policy outcome, which can be interpreted as the central planner solution. We suppose that the benevolent dictator maximizes an additive social welfare function $W(g) = u_1(g) + u_2(g)$.

As in Besley and Coate (2002), we assume that the endowments of the median voters (and of all the taxpayers) are large enough to meet their tax obligations. Differently from Besley and Coate (2002), we stress the importance of income disparity on public policy decision-making.

The efficient provision of the public good satisfies the familiar Samuelsonian condition:

$$p = (\lambda_i + \lambda_j) H'(g^e),$$

which means that the social marginal cost is equal to the social marginal benefit. The social marginal cost is the sum of the private marginal costs and it is equal to the public good price and, the social marginal benefit is the sum of the private marginal benefits$^2$.

The central planner decreases the public good provision when the price increases and increases it when preferences increase; that is: $dg^e / dp < 0$ and $dg / d\lambda_i > 0$, with $i = 1, 2$.

The optimum provision of the public good is not influenced by the way the cost is shared between regions; that is $dg / d\gamma_i = 0$, with $i = 1, 2$. From the social welfare point of view, the higher cost the median voter $i$ bears is compensated by the subsequent reduction of the relative cost for the median voter $j$. For the structure of the index $\gamma$, an increase in the relative cost for one region is always equal to the decrease in the relative cost for the other one. For this reason, the two effects always compensate each other, which means that the central planner does not care how median voters distribute the cost.

As a direct consequence of the quasi-linear preference function used in this model, the income level does not influence the central planner’s choice; i.e., $dg / dy_i = 0$, with $i = 1, 2$.

---

$^2$Solution 2 is represented in figure 2.

$^3$The proof is in appendix A.
Figure 2: The efficient outcome.
3 LEGISLATURE EQUILIBRIUM POLICY

In this section we analyse the public policy outcome when decisions are not taken by a central planner or a non-benevolent dictator, but directly by the median voters, which we consider to be the elected representatives of each region. In this case, representatives form a government and choose policy by negotiation.

We assume that, before the representatives meet in the government to choose policy, the public good is not provided. The government has to decide whether to provide the public good and how much of it to provide. If any agreement is achieved, the status quo is maintained and the government does not supply the public good. The utility that each median voter obtains in case of disagreement represents the inside option and it is $u_i^d = y_i$; with $i = 1, 2$.

In other words, if agreement is not reached, both median voters receive the utility of consuming the entire private good; that is, only private consumption exists\(^4\). An agreement is reached if the agreement utility is higher than the inside option for both players. In formulas, it must be $u_i - u_i^d > 0$, which implies $-\gamma_i pg + \lambda_i H (g) > 0$; where $i = 1, 2$.

Define with $\phi_i$ the gains from trade of median voter $i = 1, 2$, such that:

$$\phi_i = u_i - u_i^d = \lambda_i H (g) - \gamma_i pg$$

The gains from trade are equal to the private benefit minus the private cost and represent the net benefit if agreement is reached on $g$.

Representatives choose the level of the public good $g$ by bargaining. We show that by maximizing the following Nash bargaining condition:

$$\max_g \ln [-\gamma_i pg + \lambda_i H (g)] + \ln [-\gamma_j pg + \lambda_j H (g)]$$

The First Order Condition is:

$$\frac{-\gamma_i p + \lambda_i H' (g)}{-\gamma_i pg + \lambda_i H (g)} + \frac{-\gamma_j p + \lambda_j H' (g)}{-\gamma_j pg + \lambda_j H (g)} = 0$$

Now, the First Order Condition can be formulated in an alternative form, which will be very useful in the comparative statics.

Definition 1 Define with $\epsilon_i = \frac{\partial \phi_i}{\partial g} / \phi_i / g$ the elasticity of the gains from trade of player $i = 1, 2$.

The elasticity measures the percent change of the gains from trade relative to the percent change of the public good level provided by the government. The First Order Condition can be now formulated as follows:

\^4This is presented in figure 3.
Figure 3: Comparing the negotiated outcome with that of the non-benevolent dictator.
Proposition 1  The Nash Bargaining First Order Condition is satisfied if and only if the sum of the elasticities of the gains from trade is zero:

\[ \epsilon_i + \epsilon_j = 0 \]  \hspace{1cm} (3)

Proof. Any Nash Bargaining First Order Condition can be written under the following form: \( \frac{d\phi_i}{d\phi_j} + \frac{d\phi_j}{d\phi_i} = 0 \). Multiplying by \( g \) we get the sum of the elasticity of the gains from trade. \( \blacksquare \)

In other words, at the Nash bargaining equilibrium, the gains from trade elasticities of the two median voters are equal in absolute value and take opposite sign: \( \epsilon_i = -\epsilon_j \).

Proposition 3 suggests that the agreement is a compromise that lays between the representatives’ first best, as represented in figure 3 and proved in the general discussion of paragraph 4.

The Nash bargaining first order condition is used to study the comparative statics.

3.1 COMPARATIVE STATICS

3.1.1 \( \frac{dg^N}{dy_i}; \quad i = 1, 2 \) \hspace{1cm} (i)

Changes in the income of the median voters always generate conflicting interests and the bargaining outcome in the government is not straightforward. For example, each median voter would wish to consume more of the public good when his income grows up. But, at the same time he has to bear an increasing share of the public good cost. How do the median voters solve these conflicts?

Proposition 2  The government will increase the provision of the public good when the income of a median voter increases only if the elasticity of the gains from trade of the same median voter is sufficiently large and greater than a particular value expressed in the following condition:

\[ \frac{dg}{dy_i} > 0 \text{ when } \epsilon_i > \frac{H(g^*) (\lambda_j - \lambda_i) + (\gamma_i - \gamma_j) pg^*}{H(g^*) (\lambda_j + \lambda_i) - pg^*}; \quad \text{with } \epsilon_i = -\epsilon_j. \]  \hspace{1cm} (4)

Similarly, in terms of elasticity of the public good benefit, the government increases the provision of the public good when the income of a median voter increases if the elasticity of the benefit is greater than one; i.e.:

\[ \epsilon_H > 1 \]  \hspace{1cm} (5)

Proof. The proof is in the appendix B) \( \blacksquare \)

Note that \( -1 < \frac{H(g)(\lambda_j - \lambda_i) + (\gamma_i - \gamma_j) pg}{H(g)(\lambda_j + \lambda_i) - pg} < 1 \) as proved in the appendix.
THE CASE OF HOMOGENEOUS DISTRICTS  To simplify the discussion we consider the case of homogeneous districts, in which the median voters have the same preferences of the public good; i.e.: \( \lambda_i = \lambda_j \). The condition 4 becomes:

\[
\frac{d g}{d y_i} > 0 \text{ when } \epsilon_i > \frac{(\gamma_i - \gamma_j) pg^*}{2\lambda H(g^*) - pg^*}
\]

with \( \epsilon_i = -\epsilon_j \).

It is now evident how the inefficiencies come out of the conflicts generated by the income difference between the median voters. The greater the regional income disparity the greater the tax income redistribution from the wealthy region in the direction of the poorer region.

It is easy to verify that the reference ratio is zero when the median voters have the same income and relative income, which means that they share the cost equally. In this case, the government always increases the provision of the public good when total income increases, unless the elasticity of the net gains are perfectly inelastic. In the last case, it is clear that if \( \epsilon_i = 0 \) there are not net gains from increasing \( g \). This discussion leads to the following proposition:

**Proposition 3** When median voters’ income converge, the legislature is more willing to increase the provision of the public good. When the income gap increases, the government under-provides the public good, unless the gains from trade are sufficiently elastic.

Paradoxically, the case of \( \epsilon_i < \frac{(\gamma_i - \gamma_j) pg^*}{2\lambda H(g^*) - pg^*} \), which implies \( \frac{dg}{dp} < 0 \), suggests that the government may reduce the provision of public goods when the total income grows up. This effect may occur when there is a worsening of income disparity, which in turn worsens regional redistributive conflicts.

**Conclusion 1** When the economy becomes wealthier, but at the same time more unequal, the public goods provision may be reduced because of the worsening of redistributive conflicts. However, this inefficiency in the public goods provision may be compensated only by a sufficiently high elasticity of both public goods benefits and net gains.

**3.1.2 \( \frac{dg^N}{dp} \) (ii)**

We study how the central government reacts to a variation of the public good price. When policy is chosen by the central planner, he always reacts to an increase in the price by reducing the provision of the public good. Instead, when the decision is taken by the representatives of the
two districts through a bargaining process, this does not always occur. As shown in the following analysis, there may be some rare cases in which an increase of the price causes the government to increase the provision of the public good. This behavior is similar to the Giffen effect in the case of the consumption of private goods.

For example, if we consider the "general" research to be a public good, it may be the case that the government increases the financial support of it when the cost becomes too high for the private institutions.

The model states a general condition that must be satisfied in order to induce the government to increase the provision of a public good when the cost increases.

**Definition 2** Define with \( \epsilon_H = \frac{\partial H(g)/\partial g}{H(g)/g} \) the elasticity of the public good benefits.

This elasticity measures the percent change of the public good benefit relative to the percent change of the public good level provided by the government. Now, we can state the following proposition:

**Proposition 4** The legislature reacts to an increase in the price of the public good by reducing the provision. However, it will increase the provision of the public good when the elasticity of the benefit is greater than one; i.e.:

\[ \epsilon_H > 1 \]

**Proof.** The proof is in the appendix B). □

In conclusion, the effect \( dg/dp > 0 \), also known as Giffen effect, occurs when the elasticity of the public good benefits is sufficiently high, which gives an idea of the rarity of the effect. In other words, a small positive variation of \( g \) must cause a consistent gain in the public good benefit\(^5\).

3.1.3 \( dg^N/d\gamma_i; \quad i = 1, 2 \) (iii)

The study of the variation in the public good provision due to changes in the median voter’s relative income gives the same result reached above. The reason is that an increase of a median voter’s income leads to an increase of both his relative income and relative cost. In formulas:

\[
\frac{dg}{d\gamma_i} > 0 \text{ when } \epsilon_i > \frac{H\left(g^*\right)\left(\lambda_j - \lambda_i\right) + \left(\gamma_i - \gamma_j\right)pg^*}{H\left(g^*\right)\left(\lambda_j + \lambda_i\right) - pg^*} \text{ and } \epsilon_H > 1
\]

\(^5\)Similarly, in terms of elasticity of the public good benefit, the Giffen effect occurs when the elasticity of the gains from trade is greater than the following value: \( \epsilon_i > \frac{\gamma_i\phi_j + \gamma_j\phi_i}{\gamma_i\phi_j - \gamma_j\phi_i} \) with \( \epsilon_i = -\epsilon_i \). Note that \( \frac{\gamma_i\phi_j + \gamma_j\phi_i}{\gamma_i\phi_j - \gamma_j\phi_i} \geq 1 \).
where \( i = 1, 2 \) and \( \epsilon_1 = -\epsilon_2 \). The proof is in the appendix B).

However, this comparative static differs from the previous because the relative income may vary for several reasons.

It is interesting to discuss the case in which the income of median voter \( i \) remains constant, but his relative income increases because the income of median voter \( j \) decreases. In this case the analysis of the preceding paragraph still holds. The consequence is that the government can decide to increase the provision of the public good even if the income of one region is lower and the income of the other is the same as before. This looks like a paradox because the government increases the provision when the total income of the economy becomes lower; this is due to a reduction of the conflicts between regions.

Conclusion 2 *Reductions of median voters’ income may cause an increase of the public good provision when the income gap between regions reduces. This is due to a reduction of interregional redistributive conflicts.*

Summarising, interregional redistributive conflicts are more dramatic when the income gap between regions is greater. An expansion of the income gap facilitates an under-provision of public goods. The government reacts to changes in the relative income by increasing the provision of the public good only if the public good benefits and gains from trade are sufficiently elastic. Furthermore, a reduction of interregional redistributive conflicts, due to a reduction of interregional income gap, may cause an increase of public good provision even when the total income decreases.

3.1.4 \( \frac{dg^N}{d\lambda_i} \); \( i = 1, 2 \)

It is interesting to see the government reaction to a variation of the median voter preferences on the public good. An increase in the preference parameter \( \lambda \) means that the median voter receives a higher benefit from the public good. The comparative statics shows that the government increases the provision when the preference of the regional representative, who wants less of the public good, increases. Instead, the government behaviour is ambiguous when it is the preference of the representative who wants more of the public good to increase. Formally:

\[
\begin{cases}
\text{if } i \text{ wants more of } g & \Rightarrow \frac{dg}{d\lambda_i} \leq 0 \\
\text{if } j \text{ wants less of } g & \Rightarrow \frac{dg}{d\lambda_i} > 0
\end{cases}
\]

where \( i = 1, 2 \).
4 COMPARING THE FIRST BEST AND THE NEGOTIATED SOLUTION WITH THE DICTATOR’S OUTCOME

The non-benevolent dictator chooses an higher level of the public good \( g^D > g^e \) when his relative cost is lower than his relative preference; i.e.: \( \gamma_i < \frac{\lambda_i}{\lambda_j} \). This is also true when the ratio between the median voter’s \( i \) preference of the public good and the preference of median voter \( j \) is greater than the ratio between the incomes of the two representatives: \( \frac{\lambda_i}{\lambda_j} > \frac{y_i}{y_j} \).

We can use the Nash bargaining First Order Condition in the form of equation 3 in order to compare the negotiated solution with that of the non-benevolent dictator. First of all, we need to show that the marginal gain from trade of median voter \( i = 1, 2 \) is equal to \( i \)'s marginal utility:

\[
\phi'_i = \frac{\partial (u_i - u^d)}{\partial g} = -\gamma_ip + \lambda_iH'(g) = \frac{\partial u_i}{\partial g} = Mu_i \quad (7)
\]

As a consequence we obtain the following result:

**Lemma 1** The agreement lies between the two median voters’ first best outcomes, unless they coincide.

**Proof.** We need to proof that, at the agreement point, the marginal utilities of the two representatives take different sign. According to equation 7, the Nash bargaining First Order Condition 3 can be written under the following form:

\[
\frac{Mu_i}{\phi_i} = -\frac{Mu_j}{\phi_j}
\]

The sign of the elasticity of the gains from trade depends only on the sign of the marginal utilities of the representatives because the denominators are both positive by definition. Equation 3 proves that in equilibrium the elasticities of the gains from trade have different signs. This is equivalent to say that, at the agreement point, the marginal utilities of median voters take opposite signs. This proves the lemma. \( \blacksquare \)

The above result can be used to compare the bargaining with the non-benevolent dictator’s outcome.

Recall that the median voter’s first best can be interpreted as the provision of the public good that the non-benevolent dictator would choose. Indicating with \( g_i^D, g_j^D \) and \( g^{NB} \) the provision of the public good chosen respectively by the non-benevolent dictators and the negotiated provision, the three solutions are compared in figure 3. The axis measures

---

\(^6\)The proof is straightforward.
the utility of the two median voters. At the disagreement point, the median voters consume only the private good. When the public good is provided, the total utility of the median voters starts to increase. They obtain the maximum level of private utility at the private first best. If the government provides more public good than one median voter wants, his total utility decreases with respect to the first best utility. This is because the utility he gains from consuming more of the public good is lower than the utility he loses from consuming less of the private good. The agreement is a compromise, it lies between the two private first best solutions \( g^D_1 \) and \( g^D_2 \). The Nash bargaining solution represents an agreement in which one of the median voters would like to consume more of the public good and the other one would like to consume less of it.

5 INTRODUCING EXTERNALITY

In this section, we introduce externalities and study their effects on the provision of public goods. Externalities are considered differently from Besley and Coate (2003) and Alesina, Angeloni and Etro (2001a; 2001b) who consider a local public good provided non-uniformly among jurisdictions. In these cases, externalities take the form of spillovers. Instead, in this model, externalities are not spillovers. They are additional positive or negative effects caused by the national public good, different from the direct use of it, which can increase or decrease the public good benefits. An example of positive externality is the unemployment reduction due to investments in defence. We assume that externalities have the same impact in all regions.

The new utility function is:

\[
u_i = y_i - \gamma_i pg + \lambda_i H(\text{kg}) ; \quad i = 1, 2\]

where, externality is indicated with \( k \) and the public good benefit function \( H(\text{kg}) \) is any function with \( H(0) = 0, H_g(\text{kg}) > 0, H_{pg}(\text{kg}) < 0 \) and \( H_{kg}(\text{kg}) > 0 \). We exclude that the government would provide a public good which have a negative benefit for the community. For this reason we consider \( k \geq 0 \); such that:

\[
\begin{cases}
0 \leq k < 1 \Rightarrow \text{negative externalities} \\
k = 1 \Rightarrow \text{no externalities} \\
k > 1 \Rightarrow \text{positive externalities}
\end{cases}
\]

Now, we study how externalities influence the government’s outcome. Repeating the steps already done, the first order condition of the non-benevolent dictator is \( \frac{\partial u_i}{\partial g} = -\gamma_i p + \lambda_i k H'(\text{kg}) = 0 \). The optimum level of the public good \( g^D \) chosen by the dictator is the unique solution to the equation:

\[
\gamma_i p = \lambda_i k H'(\text{kg}^D_i) \quad \text{with } i = 1, 2
\]
It is clear that externalities increase the private marginal benefit when they are positive and decrease it when they are negative. It is also confirmed that when they are too negative with a value of $k \leq 0$ the private marginal benefit is negative or zero and the public good is not provided.

The comparative statics is fully confirmed: $\frac{\partial g^D}{\partial p} < 0$; $\frac{\partial g^D}{\partial \lambda_i} > 0$; $\frac{\partial g^D}{\partial \lambda_j} = 0$; $\frac{\partial g^D}{\partial \gamma_i} < 0$; $\frac{\partial g^D}{\partial \gamma_j} > 0$ and $\frac{\partial g^D}{\partial y_i} < 0$; $\frac{\partial g^D}{\partial y_j} > 0$; $\frac{\partial g^D}{\partial k} > 0$. Proofs are straightforward.

The First Order Condition that satisfies the social optimum becomes:

$$p = (\lambda_i + \lambda_j) k H'(kg)$$

Again, the public good is not provided when $k \leq 0$, positive externalities increase social marginal cost, while negative externalities decrease it.

Considering $i = 1, 2$, the comparative statics are: $\frac{dg}{dp} < 0$; $\frac{dg}{d\gamma_i} = 0$; $\frac{dg}{dy_i} = 0$; $\frac{dg}{d\lambda_i} > 0$ and $\frac{dg}{dk} > 0$. Proofs are straightforward.

In the negotiation process, externalities influence the marginal gains from trade. For each median voter, the gains from trade are maximized when the private marginal benefit of trading is equal to the private marginal cost.

It is clear that the private marginal benefit is negative when $k < 0$. For this reason, representatives would agree to provide the public good only for a $k > 0$ and such that the positive gains from trade are positive.

The First Order Nash bargaining Condition we use to represent the negotiated solution is the following:

$$-\gamma_i p + \lambda_i k H'(kg) - \gamma_j p + \lambda_j k H'(kg) = 0$$

In the comparative statics, externalities affect both conditions 6 and 5 in the same way. In particular, condition 6 becomes:

$$\frac{dg}{dp} > 0 \text{ when } \epsilon_H > \frac{1}{k} \quad (8)$$

Comparing conditions 8 and 6, we obtain the following proposition:

**Proposition 5** Positive externalities favour the Giffen effect, while negative externalities make it more difficult. Furthermore, the greater the positive externalities the higher the possibility the Giffen effect exists.

The above proposition means that, in presence of positive externalities, the government is more willing to increase the provision of a public good even when its price raises.
Similarly, condition 5 changes by the presence of externality in the following way:

\[ \frac{dg}{dy_i} > 0 \text{ when } \epsilon_H > \frac{1}{k}; \quad i = 1, 2 \]  

\textbf{Proposition 6} Positive externality facilitates the increase of the public good quantity when the income of one median voter increases. Instead, negative externalities facilitate the under-provision of the public good.

It is clear that externalities can compensate or mitigate interregional redistributive conflicts when they are positive. Conversely, they worsen redistributive conflicts when they are negatives.

\section{CONCLUSION}

The study shows that the non-benevolent dictator chooses the level of public goods that equals his private marginal cost to his private marginal benefit. However, a benevolent dictator would respect the condition of social marginal benefits equal to social marginal cost. Instead, when policy is chosen by negotiation it will be at a point in which the elasticity of regional net gains are equal and take different sign. This assures that the agreement will be a compromise between the most preferred regional representatives’ outcome.

We found that both the benevolent and the non-benevolent dictators decrease the provision of the public good when the cost increases. However, the bargaining outcome reveals that this is not always true: there may be some rare cases in which the government increases the provision of the public good when the price goes up. These cases are rare because they need a very high elasticity of both benefits and net gains in order to occur.

The paternalist dictator always increases the public good provision when at least one of the median voters increases his preference of the public good. On the contrary, the non benevolent dictator increases the provision only when his private preference increases. The bargaining approach leads to a more uncertain result. In particular, considering the median voter who has the highest preference of the public good, he will not always be able to force the government to increase the provision when he increases his preferences of the public good. However, the provision always increases when the preference of the median voter with the lowest preference increases.

Finally, the influence on the policy outcome due to variations in the income of the median voters is interesting. Both the financing of
the public good with a proportional income tax and the uniform provi-
sion across regions imply tax income redistribution, which cause inter-
regional conflicts and inefficiencies in the government policy outcome.
The non-benevolent dictator reduces the provision of the public good
when his relative income increases because his costs increase, while, he
free rides by increasing the provision when both the income and the
relative cost of the other median voter increase. However, the central
planner does not care about how the median voters share the cost.

Inefficiencies in the public good provision emerge dramatically when
we consider the negotiated policy outcome. The reason is very clear
when we consider the case of regions with homogeneous preferences. In
this case, the inefficiency is directly proportional to the inter-regional
income disparity. It disappears when the difference between regional
incomes converges to zero and becomes more relevant the larger the
income gap. This inefficiency may be compensated by a sufficiently high
elasticity of both the public good benefits and net gains.

This result leads to paradoxical conclusions. For example, when me-
dian voters become wealthier, but their income distribution is more un-
equal, the public goods provision may reduce because of the worsening
of redistributive conflicts. Similarly, reductions of median voters’ income
may cause an increase of the public good provision when the income gap
between regions reduces. This is due to a weakening of inter-regional
redistributive conflicts.

Externalities play a role in the government’s decisions. Positive ex-
ternalities facilitate the Giffen effect and mitigate inter-regional redis-
tributive conflicts, while, negative externalities have the opposite effect.
7 APPENDIX

7.1 A)

We indicate the First Order Condition with $G = -p + (\lambda_i + \lambda_j) H'(g) = 0$ and apply the implicit function theorem in order to study the comparative statics.

Proof. 

(i) $\frac{dg^c}{dp} < 0$

\[ \frac{dg}{dp} \text{ other variables constant} \equiv -\frac{G_p}{G_g} \]

\[ G_p = - \frac{\partial G}{\partial p} = -1 \]

\[ G_g = \frac{\partial G}{\partial g} = (\lambda_i + \lambda_j) H''(g) < 0 \]

\[ \frac{\partial g}{\partial p} = -\frac{G_p}{G_g} < 0 \]

Proof. 

(ii) $\frac{dg^c}{d\gamma_i}$ with $i = 1, 2$

\[ \frac{dg}{d\gamma_i} \text{ other variables constant} \equiv -\frac{G_{\gamma_i}}{G_g} \]

\[ G_{\gamma_i} = 0 \]

\[ \frac{\partial g}{\partial \gamma_i} \equiv 0 \]

Proof. 

(iii) $\frac{dg^c}{d\lambda_i} > 0$ with $i = 1, 2$

\[ \frac{dg}{d\lambda_i} \text{ other variables constant} \equiv -\frac{G\lambda_i}{G_g} \]

\[ G_{\lambda_i} = H'(g) > 0 \]

\[ \frac{\partial g}{\partial \lambda_i} \equiv -\frac{G\lambda_i}{G_g} > 0 \]

Proof. 

(iii) $\frac{dg^c}{dy_i}$ with $i = 1, 2$

\[ \frac{dg}{dy_i} \text{ other variables constant} \equiv -\frac{Gy_i}{G_g} \]

\[ G = -p + (\lambda_i + \lambda_j) H'(g) = 0, \]

\[ Gy_i = 0 \]

\[ \frac{\partial g}{\partial y_i} = 0 \]
7.2 B)

For simplicity, we denote the First Order Condition with $G = \frac{-\gamma_j p + \lambda_j H'(g)}{-\gamma_j pg + \lambda_j H(g)}$. 

**Proof.**

(i) \(\frac{\partial g^*}{\partial y_i} i=1,2\)

a) We first put the Nash Bargaining First Order Condition under the following form:

now, \(G_{yi} > 0\) when:

\[-\phi_i + g\phi'_i \phi_i^2 + \phi_j - g\phi'_j \phi_j^2 > 0\]

that gives: \(\epsilon_i > \frac{(\gamma_i-\gamma_j)pg + (\lambda_i-\lambda_j)H(g)}{\lambda_iH(g)-pg}\).

b) \(G_{yi} > 0\) when:

\[-\frac{y_i}{y_i+y_j}pg + \lambda_i H(g) + g \left( -\frac{y_i}{y_i+y_j}p + \lambda_i H'(g) \right) + \left[ -\frac{y_i}{y_i+y_j}pg + \lambda_i H(g) \right]^2 + \left[ -\frac{y_i}{y_i+y_j}pg + \lambda_j H(g) \right] - g \left[ -\frac{y_i}{y_i+y_j}p + \lambda_j H'(g) \right] - \left[ -\frac{y_i}{y_i+y_j}pg + \lambda_j H(g) \right]^2 > 0\]

that gives: \(\epsilon_H > 1\). 

**Proof.**

(ii) \(\frac{dg^*}{dp}\)

\[\frac{dg}{dp} \text{ other variables constant} = -\frac{G_p}{G_g}\]

\[G_g = \frac{\lambda_i H''(g) \left[ -\gamma_i pg + \lambda_i H(g) \right] - \left[ -\gamma_i p + \lambda_i H'(g) \right]^2 + \left[ -\gamma_i pg + \lambda_i H(g) \right]^2}{\left[ -\gamma_j pg + \lambda_j H(g) \right]^2} + \frac{\lambda_j H''(g) \left[ -\gamma_j pg + \lambda_j H(g) \right] - \left[ -\gamma_j p + \lambda_j H'(g) \right]^2}{\left[ -\gamma_j pg + \lambda_j H(g) \right]^2}\]

both ratios are negatives, which implies \(G_g < 0\).

\[G_p = \frac{\gamma_i}{\phi_i} (\epsilon_{\phi_i} - 1) + \frac{\gamma_j}{\phi_j} (\epsilon_{\phi_j} - 1)\]

\(G_p > 0\) implies: \(\epsilon_{\phi_i} > \frac{\gamma_i \phi_i + \gamma_j \phi_j}{\gamma_i \phi_i - \gamma_j \phi_j}\)

**Proof.** of the condition \(\epsilon_H > 1\)

\[G_p = \frac{-\gamma_i \phi_i + \gamma_j \phi'_i}{\phi_i^2} + \frac{-\gamma_j \phi_j + \gamma_j g \phi'_j}{\phi_j^2}\]
\[ G_p = [-H(g) + gH'(g)] \left\{ \frac{\gamma_i \lambda_i}{[-\gamma_i pg + \lambda_i H(g)]^2} + \frac{\gamma_j \lambda_j}{[-\gamma_j pg + \lambda_j H(g)]^2} \right\} \]

\( G_p > 0 \) implies \( \epsilon_H > 1. \) ■

**Proof.** (iii) \( \partial g^*/\partial \gamma_i \)

\[ \frac{dq}{d\gamma_i \text{ other variables constant}} \equiv -\frac{G_{\gamma_i}}{G_g} \]

The sign of this comparative statics depends on the sign of the derivative \( G_{\gamma_i} \).

\( a) \)

\[ G_{\gamma_i} = -\frac{p [-\gamma_i pg + \lambda_i H(g)] + pg [-\gamma_i p + \lambda_i H'(g)]}{[-\gamma_i pg + \lambda_i H(g)]^2} + \frac{p [pg (\gamma_i - 1) + \lambda_j H(g)] - pg [p (\gamma_i - 1) + \lambda_j H'(g)]}{[pg (\gamma_i - 1) + \lambda_j H(g)]^2} \]

\( G_{\gamma_i} > 0 \) \( \Rightarrow \epsilon_H > 1. \)

\( b) \)

\[ G_{\gamma_i} = p \left( \frac{\phi_i - g \phi_i'}{\phi_i^2} + \frac{\phi_j - g \phi_j'}{\phi_j^2} \right) \]

\( G_{\gamma_i} > 0 \) implies \( \epsilon_i > \frac{\phi_j - \phi_i}{\phi_i + \phi_j} \), with \(-1 < \frac{\phi_j - \phi_i}{\phi_i + \phi_j} < 1. \) By developing we obtain the standard form: \( \epsilon_i > \frac{H(g)(\lambda_j - \lambda_i) + (\gamma_i - \gamma_j)pg}{H(g)(\lambda_j + \lambda_i) - pg}. \) ■ ■

**Proof.** (iii) \( \partial g^*/\partial \lambda_i \)

\[ \frac{dq}{d\lambda_i \text{ other variables constant}} \equiv -\frac{G_{\lambda_i}}{G_g} \]

The sign of this comparative statics depends on the sign of the derivative \( G_{\lambda_i} = \frac{H'(g)\phi_i - H(g)\phi_i'}{\phi_i'}, \) which implies \( \left\{ \begin{array}{l} \text{if } i \text{ wants more of } g \Rightarrow \phi_i' > 0 \Rightarrow \frac{dq}{d\lambda_i} \leq 0 \\ \text{if } i \text{ wants less of } g \Rightarrow \phi_i' < 0 \Rightarrow \frac{dq}{d\lambda_i} > 0 \end{array} \right. \) ■

**7.3 C)**

**Proof.** (i) \( \partial g^*/\partial p \)

\[ \frac{dq}{dp \text{ other variables constant}} \equiv -\frac{G_p}{G_g} \]

\[ G_p = \frac{\lambda_i (k^2 H_{gg}(kg) [-\gamma_i pg + \lambda_i H(gk)] - [-\gamma_i p + \lambda_i H_g(gk)]^2)}{[-\gamma_i pg + \lambda_i H(gk)]^2} + \]
\[ + \lambda_j k^2 H_{gg} (kg) \left[ -\gamma_j p + \lambda_j H (kg) \right] - \left[ -\gamma_j p + \lambda_j k H_g (kg) \right]^2 \]

both ratios are negatives, which implies \( G_g < 0 \).

\[ G_p = \frac{-\gamma_i \phi_i + \gamma_i g \phi'_i}{\phi_i^2} + \frac{-\gamma_j \phi_j + \gamma_j g \phi'_j}{\phi_j^2} \]

\( G_p > 0 \) implies \( \epsilon_i > \frac{H(g)(\lambda_j - \lambda_i)(\gamma_i - \gamma_j)pg}{H(g)(\lambda_j + \lambda_i) - pg} \).  

**Proof.** of the condition \( \epsilon_H > \frac{1}{k} \)

\[ G_p = \frac{-\phi_i + g \phi'_i}{\phi_i^2} + \frac{-\phi_j + g \phi'_j}{\phi_j^2} \]

\( G_p > 0 \Rightarrow \epsilon_H > \frac{1}{k} \).

**Proof.** (iii) \( \partial g^*/\partial \gamma_i \quad i = 1, 2 \)

\[ \frac{dg}{d\gamma_i} \text{ other variables constant} \equiv - \frac{G_{\gamma_i}}{G_g} \]

The sign of this comparative statics depends on the sign of the derivative \( G_{\gamma_i} \).

a) \( G_{\gamma_i} = \frac{-p \left[ -\gamma_i pg + \lambda_i H (kg) \right] + pg \left[ -\gamma_i p + \lambda_i k H_g (kg) \right]}{\left[ -\gamma_i pg + \lambda_i H (kg) \right]^2} + \frac{p \left[ (\gamma_i - 1) pg + \lambda_j H (kg) \right] - pg \left[ (\gamma_i - 1) p + \lambda_j k H_g (kg) \right]}{\left[ (\gamma_i - 1) pg + \lambda_j H (kg) \right]^2} \)

\( G_{\gamma_i} > 0 \Rightarrow \epsilon_H > \frac{1}{k} \).

b) \( G_{\gamma_i} = p \left( -\frac{\phi_i - g \phi'_i}{\phi_i^2} + \frac{\phi_j - g \phi'_j}{\phi_j^2} \right) \)

In the case of homogenous preferences \( \lambda_j = \lambda_i \), \( G_{\gamma_i} > 0 \) implies: \( \epsilon_i > \frac{(\gamma_i - \gamma_j)pg}{2\lambda H(kg) - pg} \).  

**Proof.** (iii) \( \partial g^*/\partial \lambda_i \)

\[ \frac{dg}{d\lambda_i} \text{ other variables constant} \equiv - \frac{G_{\lambda_i}}{G_g} \]

The sign of this comparative statics depends on the sign of the derivative \( G_{\lambda_i} \).

\[ G_{\lambda_i} = \frac{k H_g (kg) \phi_i - H (kg) \phi'_i}{\phi_i^2} \]
that implies:

\[
\begin{align*}
\text{if } i \text{ wants more of } g & \Rightarrow \phi'_i > 0 \Rightarrow \frac{dg}{d\lambda_i} \leq 0 & \\
\text{if } i \text{ wants less of } g & \Rightarrow \phi'_i < 0 \Rightarrow \frac{dg}{d\lambda_i} > 0
\end{align*}
\]

**Proof.** \((iii)\) \(\partial g^* / \partial k\)

\[
\frac{dg}{dk} \text{ other variables constant} \equiv -\frac{G}{G_g}
\]

The sign of this comparative statics depends on the sign of the derivative \(G_k\).

\[
G_k = \left( \frac{\lambda_i \phi_j + \lambda_j \phi_i}{\phi_i \phi_j} \right) \left[ H_g (kg) + k^2 H_{gk} (kg) \right] - k H_k (kg) \left( \frac{\lambda_i \phi'_i}{\phi'_i^2} + \frac{\lambda_j \phi'_j}{\phi'_j^2} \right)
\]

\(G_k > 0 \Rightarrow \)

\[
\epsilon_j > \frac{(\lambda_i \phi_j + \lambda_j \phi_i) [H_g (kg) + k^2 H_{gk} (kg)] g}{(\lambda_i \phi_j - \lambda_j \phi_i) k H_k (kg)}
\]
References


