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THE IMPACT OF VERTICAL FISCAL COMPETITION ON THE TAX STRUCTURE OF A FEDERATION WITH A SYSTEM OF EQUALISATION TRANSFERS

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The impact of vertical fiscal competition on the tax structure of a federation with a system of equalisation transfers

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Abstract

This paper analyses the impact of vertical fiscal externalities on the choice of tax rates and public expenditure level by local governments operating in a federation. We assume that the federal system of taxation is represented by a linear labour income tax devoted to finance both central and local public expenditure. Thus we are going to study the regional fiscal reaction functions to changes in national tax parameters, by extending in some way the analytical procedure described in a specific previous strand of literature. In particular, in our model we assume that an equalisation system of intergovernmental transfers is working and influencing significantly the shape of the main reaction functions.

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1. Introduction

When different government levels are operating at the same time in a fiscal system, in particular when both a central and a local level are using the same tax base for acquiring revenue, vertical fiscal externalities come out and there is a fiscal competition between the two levels because tax setting decisions are interdependent[1]. In this paper we are considering the influence on

[1] In this area of research the seminal work is Gordon (1983), while clear and comprehensive surveys are in Wilson (1999) and Wellisch (2000). In the latter contribution there is also a complete analysis of the effects of households mobility on the main issues of public finance in a federal context. Janepa and Wilson (2001) have recently described a model in which both horizontal and vertical exsternalities are involved when a regional government or the national one change their tax rates.

fiscal choices by local governments of changes in national fiscal parameters, decided by a federal government.

The theoretical context to examine such a question has been recently described by several articles[2] as a game where the central government acts as a Stackelberg leader and several regional governments as followers[3]. In this paper we insert ourselves in this line of research by assuming that the federal system of taxation is represented by a linear (flat) labour income tax which is devoted to finance public expenditure, both at central and local level. Thus we are going to study the shape of the various fiscal reaction functions, by somewhat generalising the analytical procedures described in some previous papers, specifically in Besley and Rosen (1998) and Goodspeed (2000).

In this strand of literature the direct approach of broadly analysing the comparative static of the reaction functions has been avoided because too general and unable to yield tractable and conclusive results. Instead, some partial, but more interesting for the economic interpretation, analyses have been followed. For instance, as summarised by Goodspeed (2000), the reaction of local governments in response to changes in national tax parameters involves four components: an *expenditure effect* - given by the change on local public expenditure -, a *substitutability* or *complementarity effect* - due to the relationships among different tax bases at the same government level -, a *revenue effect* - related to the purpose of a local government to maintain its revenues constant - and, finally, a *deadweight loss effect* – derived from the attempt of local governments to offset the marginal disutility for their citizens.

We are going to analyse these effects in a federal context where is operating an equalisation system of intergovernmental transfers like that described in details by Dahlby and Wilson (1994) and analysed in other several theoretical contexts[4]. This scheme of transfers, applied in Canada as well as in many other countries, is based in the so called - since the seminal work by Musgrave (1961) - “fiscal capacity equalisation criterion". According to this, regions are compensated from federal revenues for the difference between a standard level of tax revenues and the revenues the regions are deemed to be able to raise if it were to apply standard tax rates to its tax bases.

This kind of formula implies, as recently explained by Sato (2000), a matching grant based on the local *tax base* and thus represents a departure from the optimal second best scheme of transfers which instead requires, to resolve tax externalities, a mix of lump-sum and matching components, with the latter based on the local *tax revenue* (or tax rate)[5]. As shown by Smart (1998),

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[2] See for instance Boadway and Keen (1996), Besley and Rosen (1998), Sato (2000), Petretto (2000) and Goodspeed (2000). In the latter paper an empirical evidence of the phenomenon has been also given.

[3] Caplan et al. (2000) analyse a symmetric view where the leadership is at local (state) level.

[4] See among others Smart (1998) and Petretto (2000).

[5] On the use of matching grants to resolve tax externalities see also Dahlby (1996).

this distortion has relevant consequences on optimal fiscal choices at local level as equalisation leads to ceteris paribus increases in local tax rates; we then show, in the following sections, that it also significantly influences the shape of reaction functions to changes of national tax parameters.

As far as the *revenue effect* is concerned we show that the sign depends on the level of the local effective tax rate versus the level of the standard one. When the former is not lower than the latter the sign is unambiguous; while, in the symmetric case, the sign depends also on the difference between the sensitivity of the effective local tax base and of the standard base (the average tax base in the federation) to changes of the national tax parameters. The sign of the *expenditure effect* with respect to change in national tax parameters can be analysed by following the same reasoning[6].

According to Besley and Rosen (1998), the *deadweight loss effect* derives from the intent of compensating the marginal disutility of a tax on a commodity which is increasing with its rate; thus we may think there is a desired gross tax rate which must be restored if one component (i.e. the central one) has been changed. However, Goodspeed (2000) has pointed out that the sign of this term depends on the objective function of the local government. With reference to a more exact definition of *deadweight loss effect,* we show that its sign actually depends on the type of *budget constraint* that a local government wants to satisfy. Even considering the same objective function we obtain two different reaction functions. Moreover, we show that, in the first of the two cases, the sign of the *deadweight loss effect* depends on the level of the effective local tax rate versus the standard one. In the second one, instead, the sign is unambiguously negative.

We finally propose a new effect related to the aim of a region to maintain constant the tax schedule which is relevant for its citizen. In other words, we are following Ahmed and Croushore (1995) in emphasising the intent by governments to look at the changes of the entire structure of tax function rather than only at a specific parameter change. We call this *tax schedule effect* and it refers to the reaction function, in response to changes of central tax parameters, for maintaining the previous tax schedule, given by the ratio of the marginal gross (central plus locale) tax rate to the average gross tax rate for a specific region (the "degree of progressivity"). We shall see, that, in response to a change of the lump sum component, the sign of the slope of this reaction function depends on the income elasticity of labour supply, i.e. of regional tax base, while, in response to a change of the tax rate, this reaction function is exactly the same as that one of the *deadweight loss effect* considered in the second case.

We begin, in section 1, by describing the model of federal taxation we are analysing. Then, in section 2, we derive the sign of the various *revenue effects*, by emphasising the role played in this respect by the equalisation grants. Section 3 is devoted to discuss *expenditure effects* and the

[6] We do not deal with the *substitutability* or *complementarity effect* because in our model we have only a tax base.

influence of the system of grants on the dimension of the absolute value of the slope of reaction functions. In section 4 we analyse the different approaches for investigating the *deadweight loss effect*. In this section we also derive the conditions for a unambiguous sign of the composite (*revenue* plus *deadweight loss*) effect of an increase of national tax rate to regional one and we investigate what we have called the *tax schedule effects*. In section 5 we conclude with a few summarising remarks.

2. The tax structure of the federation

There are n regions (i=1,..n), each one with a fixed population of N_i individuals[7]. The representative consumer in each community has preferences based on the following utility function

$$U_i = U(x_i, l_i; G, g_i) = u^i(x_i, l_i) + B(G) + b(g_i) \tag{1}$$

where $u^i(.)$ is a strictly quasi-concave sub-utility function, x_i is private consumption (which we take as the numeraire), l_i is leisure, with L_i=1-l_i as labour. G is a national public good and g_i a local one. Both are pure in nature, but the benefit of the local public good does not spill over across regions, while that of national public good accrues to all households irrespective of where they reside. The assumption of separability in the utility function implies that g and G do not affect leisure-consumption decisions of individuals. The funding of both public goods is assured by a labour income tax (a pay-roll tax).

The budget constraint for the consumer in i is given by

$$x_i = w_i L_i - T_i + I_i = Y_i + I_i - T_i \tag{2}$$

where w_i is the gross wage paid by firms in i, considered constant and in particular independent of taxation, $Y_i = w_i L_i$ is the income from labour, T_i is taxation and I_i is the lump sum income. After having substituted the following linear income tax function,

$$T_i = M + (t+\rho_i)\, Y_i \tag{3}$$

the budget constraint (2) then becomes

$$x_i = (1-\tau_i)Y_i + (I_i - M) \tag{4}$$

Here, t is the marginal tax rate defined by the federal authorities, while ρ_i is a surtax on the regional fiscal base, established by the regional authorities. Thus, $\tau_i \equiv (t+\rho_i)$ is the gross tax rate. We then adopt the idea of vertical fiscal competition: there is a joint tax base (labour income) with the regions and the federal government choosing their tax rates independently[88]. The *lump sum* component can be thought of as a tax allowance or as a universal income maintenance benefit, exclusively defined at central level: $M<0$. Therefore, the tax function is in some sense progressive across individuals of different regions: with $Y_i > Y_j$ it is $(T_i/Y_i) > (T_j/Y_j)$, as long as $\rho_i \geq \rho_j$.

The consumer i chooses consumption and leisure by maximising (1) subject to the budget constraint (4). So we obtain the following indirect utility function (the tilde denotes after-tax variables)

$$\begin{aligned} v^i &= v^i(1, w_i(1-\tau_i), I_i - M) + b(g_i) + B(G) = \\ &= \upsilon^i(\tilde{w}_i, \tilde{I}_i) + b(g_i) + B(G) \end{aligned} \qquad (5)$$

In modelling the fiscal decisions processes, we shall find useful to distinguish three types of public budget constraints.

As regards *decentralised decisions* at the regional level, we have to consider the following constraint:

$$R^i \equiv N_i \rho_i Y_i + E_i = g_i \qquad i=1,\ldots,n \qquad (6)$$

The first term of L.H.S. of (6) represents the revenue from regional taxation; the second term represents the revenue from the central government grant, which contains an equalisation aim of the type:

$$E_i = N_i e_i \qquad (7)$$

where

$$e_i = \bar{\rho}(\bar{Y} - Y_i) \qquad (8)$$

$\bar{Y}$ and $\bar{\rho}$ and are, respectively, the standard tax base and the standard regional tax rate. The former is usually chosen as a weighted average per-capita tax base, $\bar{Y} \equiv \sum_i n_i Y_i$, $\sum_i N_i = N$, $n_i = N_i / N$, while the latter could be a given reference surtax rate, established at national level, or a weighted average surtax rate, $\bar{\rho} \equiv \frac{\sum_i \rho_i n_i Y_i}{\bar{Y}}$.

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7 The features of this model are mainly derived from Boadway and Keen (1996) and Sato (2000). The mobility of people could be modelled as in the latter reference, however without significant consequences for the main conclusions of this work.

8 Actually, in many countries, a federal law establishes the range where regional governments may choose their surtax rates.

In (8) we have adopted the "full equalisation scheme"; a more flexible scheme could be represented by rewriting (8) as $e_i = \alpha\bar{\rho}(\bar{Y} - Y_i)$, where the coefficient $0<\alpha\leq 1$ shows the chosen degree of equalisation. $\alpha=1$ is the case described in (8) and adopted in the rest of the paper, if not otherwise specified.

The properties of such a system of grants have been extensively analysed in recent literature because it catches some specific features of equalisation schemes actually applied in some federal contexts[9]. It clearly works as a linear matching grant based on local tax base as we may write: $e_i = A - \bar{\rho}\, Y_i$, with $A \equiv \bar{\rho}\, \bar{Y}$.

At the federal level, we may think that the central government takes *centralised decisions* by fulfilling the following budget constraint:

$$R^f \equiv t\Sigma_i N_i Y_i + NM = G \qquad (9)$$

where there are no transfers to regions as, given (8), the sum of equalisation entitlements is cancelled out According to equation (9) the central government does not take into account the fiscal externalities of its decisions in regional budgets. If it would consider these effects it should satisfy the following social budget constraint:

$$\sum_i N_i \tau_i Y_i + NM = \sum_i g_i + G \qquad (10)$$

Equation (10), obtained by summing up all regional (6) and federal (9) budget constraints, simply says that, as in a “strictly unitary nation”, the total production costs of public goods, independently of where they are provided, are financed by overall (federal plus regional) taxation.

From (10) we may also obtain a peculiar budget constraint which, as we shall see in section 4, a region k might refer to in choosing its tax-parameters:

$$N_k \tau_k Y_k = g_k + \sum_{j\neq k} g_j + G - \sum_{j\neq k} N_j \tau_j Y_j - NM \qquad (11)$$

The L.H.S. of equation (11) is the total tax revenue yielded from the community of region k to finance overall public expenditures and tax exemptions, net of the revenues from other regions, i.e. the R.H.S. of equation (11). This constraint may be written as

$$N_k \tau_k Y_k = g_k + \Phi_k \qquad (12)$$

[9] See for instance Dahlby and Wilson (1994), Smart (1998) and Petretto (2000).

where Φ_k represents all the public expenditures and taxes which are exogenous as far as the decisions of region *k* are concerned.

For later reference we consider, as the relevant tax schedule for region *k,* the "degree of progressivity" of the revenue included in (12), measured by the ratio of the marginal tax rate to the average tax rate[10]:

$$\Pi_k = \frac{\tau_k}{\dfrac{N_k \tau_k Y_k + N_k M}{N_k Y_k}} = \frac{1}{1 + \dfrac{M}{\tau_k Y_k}} \tag{13}$$

Therefore this measure of the "degree of progressivity" clearly depends on the level of national fiscal parameters, *M* and *t*, as well as on the level of the regional tax rate ρ_k.

We model the interactions between the two-levels governments as a Stackelberg game, where the central government acts as the leader and each region as a follower. Therefore, central government makes its strategies (the fiscal parameters) by taking into account each region's reaction function and each region chooses its fiscal parameters by considering the fiscal parameters at the central level and of the other regions as given. In effect, we presume that there are enough regions so that each may ignore the effects of its decisions on other regions, as well as on the federal government.

Thus we have that region *k* chooses (g_k, ρ_k) in order to maximise a regional welfare function $V_k(.)$, given a budget constraint like (6) or (12) and given the fiscal national parameters *t,M,G* and of others regions g_j, ρ_j all $j \neq k$[11]. From the solution of the F.O.C.s of this program we can derive the region *k* optimal policy variables as reaction functions: $g_k = g_k(G, t, M; g_j, \rho_j \ \forall j \neq k)$ and $\rho_k = \rho_k(G, t, M; g_j, \rho_j \forall j \neq k)$. Consequently the behaviour of the central government is to chose its fiscal parameters *G*, *t,* and *M* by maximising an objective function[12], subject to budget constraint (9) and all regional reaction functions.

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[10] See Ahmed and Croushore (1995) for an analysis of the impact of such a policy. Of course the policy of maintaining progressivity should be even more meaningful in a context of differentiated individuals within each region.

[11] Actually a regional government chooses one of these two (e.g. ρ_k) and the other (g_k) is determined by the budget constraint (6) or (12).

[12] As in Petretto (2000) we may think at a generic objective function as $V_f = \Omega(G, t, M)$, with $\Omega_G > 0$, $\Omega_t < 0$, $\Omega_M < 0$. This objective functional form is quite general and may include several well-known traditional specifications. One could be the *welfarist-hierarchical* welfare function: $W = W(V_1, \ldots V_n)$, with $W_i > 0$. Another specification could suggest that $\Omega(G, t, M)$ is an indirect utility function of a representative consumer, who can be regarded as the median voter of the whole population and who is decisive in choosing federal tax policies under majority rule. This objective function could also be the result of a bargaining between, say, politicians and national Trade Unions.

In order to examine the changes in regional tax parameters induced by changes in national tax parameters in the literature the direct approach of broadly analysing the shape of functions like $g_k(.)$ and $\rho_k(.)$ has been avoided because too general and not conclusive. Instead, more interesting, for the economic interpretation, partial analyses have been followed. For instance, as recently summarised by Goodspeed (2000) the reaction of local government may involve four components: an *expenditure effect*, a *substitutability* or *complementarity effect*, a *revenue effect* and a *deadweight loss effect*[13].

In the following sections we are going to analyse these effects and some new ones according to the model just described[14].

3. The Revenue effect on regional fiscal parameters

3.1. The Revenue effect on regional tax rates (RET)

Firstly, we may give the following

Definition

The revenue effect –RET- is given by the change in the regional surtax rate in response to changes in national tax parameters in order to maintain constant the regional revenue level according to budget constraint (6).

From (6), (7) and (8) the level of regional tax revenue is given by

$$R^k = N_k \rho_k Y_k + N_k \overline{\rho}(\overline{Y} - Y_k) = N_k(\rho_k - \overline{\rho})Y_k + N_k \overline{\rho}\,\overline{Y} \tag{14}$$

On computing the sign of this revenue effect we make this set of assumptions:

Assumption 1: (i) both the standard regional tax rate and the standard tax base are invariant to each regional fiscal decision; (ii) the standard regional tax rate is invariant to national fiscal decisions while the standard tax base is not; (iii) in each region: a) leisure is a normal good, ($\partial L_k/\partial M>0$); b) the supply of labour schedule is increasing with respect to post-tax wage: ($\partial L_k/\partial \widetilde{w}_k>0$).

Assumption 1(i) is in some sense linked to the idea that the central government faces many small regions which take their fiscal decisions independently. Assumption 1(ii) is stronger because, if we adopt the definition of the standard tax rate as an average rate, a change on it actually occurs when national tax parameters change, even if we may think it negligible. Assumptions 1(iiia) and (iiib) are usually made in this kind of literature.

[13] The first two effects of this comparative static have been derived by Boadway and Keen (1996), the last two effects have been treated in Besley and Rosen (1998) and also in Goodspeed (2000).

First let us consider the reaction of region *k* facing an increase of lump sum component *M* (a reduction of the tax allowance) decided by central government.

The change on its own tax rate must solve this equation

$$R_M^k dM + R_\rho^k d\rho_k = 0 \tag{15}$$

From this we may derive the following *revenue effect*:

$$\left(\frac{d\rho_k}{dM}\right)_{RE} \equiv -\frac{R_M^k}{R_\rho^k} \tag{16}$$

Proposition 1:

(i) *In region k, where* $\rho_k \geq \overline{\rho}$, $\left(\frac{d\rho_k}{dM}\right)_{RE}$ <0

(ii) (ii) *In region k, where* $\rho_k < \overline{\rho}$, $\left(\frac{d\rho_k}{dM}\right)_{RE}$ <0 *if* $\frac{\partial \overline{Y}}{\partial M} > \frac{\partial Y_k}{\partial M}$.

Proof

From (14) we firstly we have

$$R_M^k \equiv N_k [(\rho_k - \overline{\rho})(\partial Y_k/\partial M) + \overline{\rho}(\partial \overline{Y}/\partial M)] = N_k \rho_k(\partial Y_k/\partial M) + N_k \overline{\rho}[(\partial \overline{Y}/\partial M) - (\partial Y_k/\partial M)] \tag{17}$$

Then

$$R_\rho^k \equiv N_k [Y_k + (\rho_k - \overline{\rho})(\partial Y_k/\partial \rho_k)] = N_k [Y_k - (\rho_k - \overline{\rho}) w_k(\partial L_k/\partial \widetilde{w}_k) w_k] \tag{18}$$

Given Assumption 1(ii) and (iiia), we have $\partial Y_k / \partial M > 0$ and $\partial \overline{Y} / \partial M = \sum_i n_i (\partial Y_i / \partial M) > 0$; therefore with $\rho_k \geq \overline{\rho}$, $R_M^k > 0$. Moreover, given Assumption 1 (ii) and (iiib), and the fact that ρ_k is the lowest rate consistent with the revenue requirement [Besley and Rosen 1998], it is also $R_\rho^k > 0$. Therefore (i) is proven. With $\rho_k < \overline{\rho}$, from the second part of (17), it is $R_M^k > 0$ if $\partial \overline{Y} / \partial M > \partial Y_k / \partial M$, and thus also (ii) is proven.□

An increase of the national lump sum component of income tax tends to increase, through an income effect on labour income, the regional tax bases. Thus, according to Proposition 1, the reaction by the revenue effect of a region, with a local tax rate not less than the standard one, to an increase of the national lump sum tax leads, because of an increase of the total regional revenue (own taxes plus transfers), to a reduction of the local tax rate. The same sign of the slope of the reaction function occurs when the local tax rate is less than the standard one if the increase of

[14] As said, in our model there is no room for a *substitutability* or *complementarity effect* because we have only a tax

regional tax base (regional income) to the lump sum tax is less than the increase of the standard tax-base (average income). In fact, from (17) we see that also in this case the total regional revenue is increased. Note however that, with $\rho_k < \overline{\rho}$ and $\partial\overline{Y} / \partial M < \partial Y_k / \partial M$, it may well happen that region k is induced to increase its tax rate. In this case, actually, the effect on the total regional revenue of the change in the national tax parameter is, given the equalisation grant, uncertain.

Let us now consider the reaction of region k facing an increase of t decided by central government. The change on its own tax rate must solve this equation

$$R_t^k dt + R_\rho^k d\rho_k = 0 \tag{19}$$

and then we define the following revenue effect

$$\left(\frac{d\rho_k}{dt}\right)_{RE} \equiv -\frac{R_t^k}{R_\rho^k} \tag{20}$$

Proposition 2:

(i) *In region k, where* $\rho_k \geq \overline{\rho}$, $\left(\frac{d\rho_k}{dt}\right)_{RE} > 0$

(ii) *In region k, where* $\rho_k < \overline{\rho}$, $\left(\frac{d\rho_k}{dt}\right)_{RE} > 0$ *if* $\left|\frac{\partial\overline{Y}}{\partial t}\right| < \left|\frac{\partial Y_k}{\partial t}\right|$

Proof

Now we have

$$R_t^k \equiv N_k\,[\,(\rho_k - \overline{\rho})\,(\partial Y_k/\partial t) + \overline{\rho}\,(\partial\,\overline{Y}/\partial t)] = N_k\,\rho_k(\partial Y_k/\partial t) + N_k\,\overline{\rho}\,[(\partial\,\overline{Y}/\partial t)-(\partial Y_k/\partial t)] \tag{21}$$

Given Assumption 1 (iiib), we have $\partial Y_k / \partial t = \partial Y_k / \partial \rho_k = -w_k(\partial L_k / \partial \tilde{w}_k) w_k < 0$ and $\partial\overline{Y} / \partial t = \sum_i n_i(\partial Y_i / \partial t) < 0$; therefore, with $\rho_k \geq \overline{\rho}$, $R_t^k < 0$, while R_ρ^k is clearly positive as before. Thus (i) is proven. With $\rho_k < \overline{\rho}$, from the second part of (21), $R_t^k < 0$ if $\partial\overline{Y} / \partial t > \partial Y_k / \partial t$, and thus also (ii) is proven.□

An increase of the national tax rate of income tax tends to reduce, through a substitution effect greater than the income one on labour income, the regional tax bases. Thus, according to Proposition 2, the reaction by the revenue effect of a region, with a local tax rate not lower than the standard one, to an increase of the national tax rate leads, because of a reduction of regional revenue, to an increase of the local tax rate. The same sign of the reaction function occurs when the local tax rate is lower than the standard one if the decrease of regional the tax base (regional

base.

income) to the national tax rate is greater than the decrease of the standard tax base (average income), because also in this case the equalisation is not able to compensate the shrinkage of the regional tax base and thus the total regional revenue is reduced. However note, as before, that with $\rho_k < \overline{\rho}$ and $\left|\frac{\partial \overline{Y}}{\partial t}\right| > \left|\frac{\partial Y_k}{\partial t}\right|$ it may well happen that region *k* desires to reduce its tax rate, because of a somewhat counterintuitive increase in total regional revenue due to the working of the equalisation scheme.

3.2. The Revenue effect on regional public expenditure (REE)

We start by this

Definition

The expenditure effect - REE - is the change in regional public expenditure level in response to changes in national tax parameters in order to compensate the regional revenue change according to budget constraint (6)

From (6) we have

$$R_M^k dM - dg_k = 0 \tag{22}$$

and

$$R_t^k dt - dg_k = 0 \tag{23}$$

Therefore we have the following

Proposition 3

(i) *In region k, where* $\rho_k \geq \overline{\rho}$, $\left(\frac{dg_k}{dM}\right)_{RE} = R_M^k > 0$, $\left(\frac{dg_k}{dt}\right)_{RE} = R_t^k < 0$

(ii) *In region k, where* $\rho_k < \overline{\rho}$, $\left(\frac{dg_k}{dM}\right)_{RE} = R_M^k > 0$, $\left(\frac{dg_k}{dt}\right)_{RE} = R_t^k < 0$, *if respectively* $\frac{\partial \overline{Y}}{\partial M} > \frac{\partial Y_k}{\partial M}$ *and* $\left|\frac{\partial \overline{Y}}{\partial t}\right| < \left|\frac{\partial Y_k}{\partial t}\right|$.

Proof

See proofs of Proposition 1 and 2 , as far as the sign of R_M^k and R_t^k is concerned. □

The interpretation of this Proposition follows that of the previous two ones. There is in fact a strict link among the relevant revenue effects, as we may write $\left(\frac{d\rho_k}{d\theta}\right)_{RE} R_\rho^k \equiv -\left(\frac{dg_k}{d\theta}\right)_{RE}$, with respectively *θ=M,t.*

An interesting question is to ascertain if the absolute value of the slope of regional reaction functions, for maintaining the revenue constant, to changes in national tax parameters

$\left|\left(\frac{d\sigma_k}{d\theta}\right)_{RE}\right|$ - where $\sigma_k = \rho_k, g_k$, and *θ=M,t.*

is influenced and in which direction by the system of equalisation grants. Actually, while we may say nothing of transparent and conclusive as far as the slopes $\left|\left(\frac{d\rho_k}{d\theta}\right)_{RE}\right|$ are concerned, we may quickly ascertain the following result about the slope of the reaction functions of regional expenditure.

Proposition 4

The slope of both the two reaction functions, $\left|\left(\frac{dg_k}{d\theta}\right)_{RE}\right|$, *where θ=M,t - is increased by the system of equalisation grants if respectively* $\frac{\partial \overline{Y}}{\partial M} > \frac{\partial Y_k}{\partial M}$ *and* $\left|\frac{\partial \overline{Y}}{\partial t}\right| < \left|\frac{\partial Y_k}{\partial t}\right|$.

Proof

In order to prove this Proposition it convenient to consider a flexible formula of equalisation grants which may allow also for a partial equalisation. In other words, we should rewrite (8) as $e_i = \alpha\overline{\rho}(\overline{Y} - Y_i)$, where the coefficient $0<\alpha\leq 1$ shows the chosen degree of equalisation. Elsewhere in the paper we take into account the full equalisation case α=1; of course if, instead, were $\alpha=0$ there would be no equalisation at all. So what we need to ascertain is if $\left|\left(\frac{dg_k}{d\theta}\right)_{RE}\right|$ are increasing with α. In this new context (17) and (21) become, respectively

$$R_M^k = N_k\, \rho_k(\partial Y_k/\partial M) + N_k\, \alpha\overline{\rho}\, [(\partial \overline{Y}/\partial M) - (\partial Y_k/\partial M)] \tag{17'}$$

$$R_t^k \equiv N_k\, \rho_k(\partial Y_k/\partial t) + N_k\, \alpha\overline{\rho}\, [(\partial \overline{Y}/\partial t) - (\partial Y_k/\partial t)] \tag{21'}$$

Therefore we have $\frac{\partial \left| R_{\theta}^{k} \right|}{\partial \alpha} > 0$, i.e. the absolute value of the slope of the two reaction functions is increasing with the degree of equalisation, if respectively $(\partial \overline{Y}/\partial\theta)-(\partial Y_k/\partial\theta)>0$.□

Thus, according to Proposition 4, the "strength" by which a local government reacts, by modifying own public expenditure level to the changes of national fiscal strategies, is greater if the change in regional tax base due to the change in national tax parameters is greater than that of the standard tax base (average income). In this case, in fact, the equalisation system of grants is not able to exactly compensate the effects on regional tax base.

4. The deadweight loss effect and the tax-schedule effect on regional tax rates

According to Besley and Rose (1998) the *deadweight loss effect* derives from the aim to reduce the marginal disutility of a tax on a commodity (like leisure/labour in our case) "which is increasing with its rate, ceteris paribus"; so we may think there is a desired gross tax rate which must be restored if one component (i.e. the central one) has been is changed. Goodspeed (2000) pointed out that the sign of this term depends on the "objective function of the local government, particularly on whether the local government considers only the local excess burden or the gross (national plus local) excess burden in setting the local tax rate".

In this section we are going to give a more exact definition of *deadweight loss effect* and to show that its sign actually depends on *the budget constraint* that a local government wants to satisfy. Even considering only one objective function, we obtain two measures of the reaction function, as conjectured by Goodspeed (2000). Moreover, we show that, according to our model, in the first of the two cases, the sign of the *deadweight loss effect* depends on the level of the effective local tax rate with respect to the standard one.

Now we have this

Definition

The deadweight loss effect – DLET- is given by the change in regional tax rate in response to changes in the national one in order to maintain constant the regional tax burden, measured by an "appropriate" marginal cost of public funds

We may assume that a region *k,* in choosing own optimal tax-parameters, is going to maximise a utilitarian welfare function like the following:

$$V_k = N_k \upsilon^k (\widetilde{w}_k, \widetilde{I}_k) + N_k (B(G) + b(g_k)) \,. \tag{24}$$

However, we have to distinguish two case, according to which budget constraint a region wants to satisfy; (1) the regional one or (2) the overall budget, but still considering as given the choices of fiscal parameters by the other regions and the central government. Consequently two concepts of marginal cost of public funds become relevant for region *k*, say a "regional" one and a "gross" one.

3.1. The deadweight loss effect (DLET): Case 1

Region *k* chooses ρ_k in order to maximize

$$N_k \upsilon^k(\widetilde{w}_k, \widetilde{I}_k) + N_k(B(G) + b(R^k)) \tag{25}$$

given the fiscal national parameters *t,M,G* and of others regions g_j, ρ_j all $j \neq k$. Note that the argument of *b*(.) in (25) derives from budget constraint (6).

The F.O.C. with respect to ρ_k^* gives the following equation:

$$\partial \upsilon^k / \partial \rho_k + b'[N_k w_k L_k(.) + N_k(\rho_k - \overline{\rho})\, w_k(\partial L_k / \partial \rho_k)] = 0 \tag{26}$$

By Roy's identity $\partial \upsilon^k / \partial \rho_k = -\ \upsilon_I^k\, w_k L_k$, so we obtain:

$$-\,\upsilon_I^k L_k + N_k b'[L_k - (\rho_k - \overline{\rho})\, w_k(\partial L_i / \partial \widetilde{w}_k)] = 0. \tag{27}$$

By denoting

$$\gamma_k \equiv [(\rho_k - \overline{\rho})/(1-\tau_k)]\varepsilon_{Lk} \tag{28}$$

where $\varepsilon_{Lk} = (\partial L_k / \partial \widetilde{w}_k)(\widetilde{w}_k / L_k)$ is the elasticitiy of the supply of labour, from (27) we obtain the condition of equality of marginal benefit of public funds invested in the local public good with the regional marginal cost of public funds [Smart 1998 and Petretto 2000]:

$$MBPF^k \equiv N_k \frac{b'}{\upsilon_I^k} = \frac{1}{1-\gamma_k} \equiv MCPF^k. \tag{29}$$

Let us define with $\left(\dfrac{d\rho_k}{dt}\right)_{DL}^k$ the change in regional surtax rate to an increase of national tax rate which maintains constant $MCPF^k$; moreover, as in Besley and Rosen (1998), we make, for simplicity, this further assumption:

Assumption 2: the elasticity of the supply of labour in region *k* is constant

Thus we have the following

Proposition 5

(i) *In region k, where* $\rho_k > \overline{\rho}$, $\left(\frac{d\rho_k}{dt}\right)_{DL}^{k}$ <0;

(ii) *In region k, where* $\rho_k < \overline{\rho}$, $\left(\frac{d\rho_k}{dt}\right)_{DL}^{k}$ >0

Proof

For maintaining $MCPF^k$ constant, given Assumption 2, we have only to maintain constant the expression $[(\rho_k - \overline{\rho})/(1-\tau_k)]$. Therefore by directly computing the derivative of the implicit function we get:

$$\left(\frac{d\rho_k}{dt}\right)_{DL}^{k} = -\frac{\rho_k - \overline{\rho}}{1-(t+\overline{\rho})} \tag{30}$$

and so both parts of the proof are straightforward. □

Therefore, the sign of the *deadweight loss effect*, according to the interpretation of *Case 1*, where regions try to maintain constant the social cost of the regional taxation only, depends on the level of regional tax rate with respect to level of the standard one.

Interesting enough is to combine Proposition 2(ii) with Proposition 5(ii), obtaining this

Proposition 6

In region k, where $\rho_k < \overline{\rho}$ *and* $\left|\frac{\partial \overline{Y}}{\partial t}\right| < \left|\frac{\partial Y_k}{\partial t}\right|$, *the total reaction, for both the revenue and the deadweight loss effects, to an increase in t leads to an increase in the regional surtax* ρ_k.

The proof is clearly straightforward.

As far as the absolute value of the slope of this reaction function, now, we have the following

Proposition 7

The slope of the reaction function $\left|\left(\frac{d\rho_k}{dt}\right)_{DL}^{k}\right|$ *is increased by the system of equalisation grants*

Proof

Also for proving this Proposition, as for Proposition 4, is useful to consider the case of flexible degree of equalisation: $e_i = \alpha\bar{\rho}(\bar{Y} - Y_i)$. Now (30) becomes:

$$\left(\frac{d\rho_k}{dt}\right)_{DL}^{k} =- \frac{\rho_k - \alpha\bar{\rho}}{1-(t+\alpha\bar{\rho})} \tag{31}$$

Therefore we need just to show that $\left|\left(\frac{d\rho_k}{dt}\right)_{DL}^{k}\right|$ is increasing with degree of equalisation α. Actually, by differentiating (31) with respect to α, we get $-\frac{-[1-(t+\alpha\bar{\rho})]+(\rho_k - \alpha\bar{\rho})}{[1-(t+\alpha\bar{\rho})]^2} = \frac{1-\tau_k}{[1-(t+\alpha\bar{\rho})]^2} > 0$. □

3.2. The deadweight loss effect (DLET): Case 2

Region k now chooses ρ_k in order to maximise:

$$N_k \upsilon^k(\tilde{w}_k, \tilde{I}_k) + N_k(B(G) + b(N_k \tau_k Y_k - \Phi_k)) \tag{32}$$

given budget constraint (12) and the fiscal national parameters t,M,G and of others regions g_j, ρ_j all $j \neq k$}. Note that the argument of $b(.)$ in (32) now derives from budget constraint (12).

The F.O.C. with respect to ρ_k^* gives the following equation:

$$-\upsilon_I^k L_k + N_k b'[L_k - \tau_k w_k(\partial L_i / \partial \tilde{w}_k)]=0 \tag{33}$$

and then condition

$$MBPF^k \equiv N_k \frac{b'}{\upsilon_I^k} = \frac{1}{1-\gamma_{Gk}} \equiv GMCPF^k \tag{34}$$

where, now, it is $\gamma_{Gk} \equiv \frac{\tau_k}{(1-\tau_k)}\varepsilon_{Lk}$.

Equation (34) gives the equality of marginal benefit of public funds with the “gross” marginal cost of public funds. Thus we may refer to a "gross" reaction function for measuring the deadweight loss effect from the purpose of maintaining constant the gross marginal cost of public funds, $GMCPF^k$. In fact we have the following

Proposition 8

$$\left(\frac{d\rho_k}{dt}\right)_{DL}^{Gk} =-1$$

Proof

For maintaining γ_{Gk}, and then $GMCPF^k$, constant to a change on *t*, given Assumption 2, we must simply maintain constant τ_k, by compensating the increase with a symmetric reduction of ρ_k. □

Therefore, the sign of the deadweight loss effect, according to the interpretation of *Case 2*, where regions want to maintain constant the social cost of the whole taxation yielded in the same region, is certainly negative and it is independent on the relative level of the regional tax-rate

It is easy to verify that we may design a link between the two concepts of marginal cost of public funds - the gross, $GMCPF^k$, and the regional one, $MCPF^k$ – and thus of the two reaction functions of the *Case 1* and *Case 2*. In fact we have that $\gamma_{Gk} = \gamma_k + \frac{t+\bar{\rho}}{1-\tau_k}\varepsilon_{Lk}$, and then we may directly compute the following relationship between the two reaction functions:

$$\left(\frac{d\rho_k}{dt}\right)_{DL}^{Gk} = \left(\frac{d\rho_k}{dt}\right)_{DL}^{k} [1-(t+\bar{\rho})]-[1-(\rho_k-\bar{\rho})] \qquad (35)$$

Therefore, by substituting equation (31) in R.H.S. of (35), we get once again Proposition 8.

3.3. The tax schedule effect (TSET)

A region could be also interested to avoid that a change in national tax parameters can change its tax schedule, by restoring for instance the degree of progressivitiy which was prevailing before the change. We thus refer to the following

Definition
The tax-schedule effect – TSET- is given by the change in regional tax rates in response to changes in national tax parameters in order to maintain the degree of progressivity of the regional tax structure measured by index Π_k of (13).

Therefore we are going to establish the sign of the two slopes $\left(\frac{d\rho_k}{dM}\right)_{TSET}^{k}$ and $\left(\frac{d\rho_k}{dt}\right)_{TSET}^{k}$, by means of this

Proposition 9

(i) $\left(\frac{d\rho_k}{dM}\right)_{TSET}^{k}$ >(=,<) 0 if $\xi_k \equiv (\partial Y_k/\partial M)(M/Y_k)$<(=,>) 1;

(ii) $\left(\frac{d\rho_k}{dt}\right)_{TSET}^{k}$ = -1

Proof

It results from (13) that if a region k wants to maintain Π_k constant, in response to a change in M and t, it should change the tax rate ρ_k for maintaining constant the term $(M/\tau_k Y_k)$. Thus by implicit differentiation we get slope of reaction function:

$$\left(\frac{d\rho_k}{dM}\right)_{TSET}^{k} = -\frac{\tau_k Y_k - M\tau_k(\partial Y_k/\partial M)}{-Y_k - \tau_k(\partial Y_k/\partial\rho_k)} = \frac{\tau_k Y_k(1-\xi_k)}{Y_k + \tau_k(\partial Y_k/\partial\rho_k)}$$

As the denominator is positive for the same reasoning by which it turns out that $R_\rho^k > 0$, part (i) is proven[1514]. The proof of part (ii) is also straightforward if we take into account that the change of Y_k to a change of t is the same of that due to a change in ρ_k. Thus, for maintaining constant $(M/\tau_k Y_k)$ with respect to a change in t, we have only to maintain constant the gross tax rate τ_k, and then it is clearly $\left(\frac{d\rho_k}{dt}\right)_{TSET}^{k}$ = -1. □

From (i) of proposition 9 it results that a region k which wants to maintain the degree of progressivity it should reduce (increase) its tax rate in response to an increase of M if its tax base is very (not very) elastic to changes of lump sum tax component. From (ii), it results that $\left(\frac{d\rho_k}{dt}\right)_{TSET}^{k} = \left(\frac{d\rho_k}{dt}\right)_{DL}^{Gk}$, i.e. what we have called the *tax schedule effect* on the regional tax rate of an increase of the national tax rate is exactly equal to the *deadwheight loss effect* of Case 2, when a region k wants to maintain constant the "gross" marginal cost of public funds.

[15] It is easy to verify how the elasticity of tax base to tax allowance is related to income elasticity of labour supply, as we have:

$$\xi_k = -\frac{M}{(I_k - M)}\frac{\partial L_k}{\partial(I_k - M)}\frac{(I_k - M)}{L_k}$$

5. Summarising and concluding remarks

In order to investigate on the changes in regional tax parameters induced by changes in national tax parameters we look at a model of fiscal federalism which can be easily referred to a specific strand of literature, recently summarised for instance by Goodspeed (2000). In particular, we consider the reaction of local governments involving a *revenue effect*, an *expenditure effect*, and a two different *deadweight loss effect*. Moreover, we add a new effect which we call as *tax schedule effect*.

We model a federal economy as a Stackelberg game with the leadership at the central level and several regions acting as followers. The federal system of taxation is assumed to be represented by a linear (flat) labour income tax which feeds public expenditure, both at central and local level. Thus we have a classical problem of vertical fiscal competition between two government levels. In this standard theoretical context we add a system of equalisation grants of the type adopted in Canada, which is able to interfere in a significant way with the regional strategies of reaction to central government moves. Such a system of grants tends to equalise the so called regional "fiscal capacity", as it is linked to the difference between the effective regional tax base and a standard (average) tax base which a standard tax rate is applied to.

An increase of the national lump sum component of income tax (a decrease of a tax allowance) tends to increase, through an income effect on labour supply, the regional tax bases. According to Proposition 1, the reaction by the *revenue effect* of a region, with a local tax rate not lower than the standard one, to an increase of the national lump sum tax leads to a reduction of the local tax rate. In this case we would certainly have an increase of the total regional revenue (from own taxes plus grants) which may easily allow for a decrease of local tax effort and burden. The same sign of the reaction function occurs when the local tax rate is lower than the standard one if the increase of regional tax base (regional income) to the lump sum tax is less than the increase of the standard tax base (average income of the federation). In fact, also in this case the total regional revenue is certainly increased by the national fiscal change.

The increase of regional tax revenue is the reason why, according to Proposition 3, an increase of the national lump sum component of income tax tends to increase for sure the local public expenditure in regions where the local tax rate is not lower than the standard one. The same sign of this *expenditure effect* will occur in regions where the local tax rate is less than the standard one if the increase of regional tax base to the lump sum tax is less than the increase of the standard tax base.

An increase of the national tax rate of the income tax tends to reduce the regional tax bases through a substitution effect on labour supply prevailing on the income one. Therefore, according to

Proposition 2, the reaction by the *revenue effect* of a region, with the local tax rate not lower than the standard one, to an increase of the national tax rate leads, because of a reduction of regional tax revenue, to an increase of the local tax rate. The same sign of the reaction function occurs when the local tax rate is lower than the standard one if the decrease of the regional tax base to the national tax rate is greater than the decrease of the standard tax base, because also in this case the total regional revenue is reduced. Therefore, regions must react to a reduction of own total revenue with an increase of the regional tax effort and burden.

Following a similar reasoning we may affirm, with Proposition 3, that an increase of the national tax rate of income tax tends to reduce (surely in regions where the local tax rate is not lower than the standard one and with the same previous condition in the others) the regional public expenditure.

We have also investigated the question whether the absolute value of the slope of the reaction functions of a region, for maintaining the revenue constant, to changes in national tax parameters is influenced and in which direction by the "fiscal capacity-equalisation" system of grants. According to Proposition 4, the sensitivity by which the local government reacts, by modifying the level of regional public expenditure to the changes on national fiscal strategies, is greater if the change in regional tax-base due to the change in national tax parameters is greater than that of the standard tax base. In this case, in fact, the equalisation system of grants is not able to exactly compensate the effects on regional tax base. Unfortunately, no similar specific and conclusive propositions can be proved about the absolute value of the slope of revenue effects on local tax rates.

As far as the *deadwheight loss effect* is concerned, we have specifically referred to it as the result of the attempt by regions to offset the changes on the marginal cost of public funds, a measure of the local tax burden. Therefore, we have distinguished two cases, according to which budget constraint a regional government wants to or is obliged to satisfy: the regional budget or the overall budget, but still considering as given the choices of fiscal parameters by the other regions and the central government. We may say that the second case is more realistic as one would think that citizens would be concerned with their overall level of taxation. In any case, in the first hypothesis it is the level of regional own taxes that is relevant as tax burden; in the second one it is, instead, the level of overall distortionary taxation (regional plus central) which is relevant. Consequently two concepts of marginal cost of public funds arise.

In Case 1, where regions try to maintain constant the social cost of only regional taxation - what we call the *"regional"* marginal cost of public funds - the sign of the *deadweight loss effect* depends on the level of regional tax rate with respect to level of the standard one. In particular, according to Proposition 5, it is positive in regions where the local tax rate is lower than the standard one and *viceversa* for the others. In Case 1, we reach, with Proposition 6, probably the strongest result of the paper: in regions where the local tax rate is lower than the standard one and the decrease of regional the tax base to the national tax rate is greater than the decrease of the standard tax-base, both *revenue* and *deadwheight loss effects* leads to an increase in regional tax rate. In other wards, it is in these regions that an increase of national tax rate is followed for sure by an increase of regional fiscal effort. With Proposition 7, we ascertain that the absolute value slope of the reaction function due to this *deadwheight loss effect* is surely increased by the adopted system of equalisation grants.

In Case 2, where regions try to maintain constant the social cost of the whole taxation yielded in the same region - what we call the *"gross"* marginal cost of public funds -, the sign of the *deadweight loss effect* is, by Proposition 8, certainly negative and it is independent on the relative level of the regional tax-rate.

Finally, by Proposition 9 we ascertain that, if a region wants to maintain constant the tax schedule, i.e. the ratio of the marginal tax rate to the average rate tax rate, it has to increase (reduce) its tax rate in response to an increase of the national lump sum component, if the elasticity of its tax base is lower (higher) than 1, and it has to reduce its tax rate in response to an increase of the national one. In the latter case we see as the *tax schedule effect* is equivalent to the Case 2 *deadweight loss effect.*

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