

Immigration and PAYGO pension systems in the presence of increasing life expectancy

Giuseppe Di Liddo^a

^a*University of Bari, DISAG and University of Salento, Dipartimento di Scienze dell'Economia
e-mail: giudiliddo@gmail.com*

Abstract

Through an Overlapping Generations Model, this paper studies the effects of migratory flows on a pay-as-you-go (PAYGO) pension system in the presence of linear increase in life expectancy over time.

Our results show that immigration is likely to induce distributional conflicts between different groups in society. The increasing number of contributors due to immigration will result in higher pension benefits for both retirees and older workers. Future immigrants' pension claims imply that younger workers will either gain or lose from immigration depending on the immigrants' labour productivity.

In the case of small increases in life expectancy, immigration increases the welfare of the majority of population, given by retirees and older workers. On the other hand, in the presence of high increases in life expectancy, immigration may affect negatively the welfare of the majority of population in the absence of further parametric reforms of the pension system.

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JEL Classification: F22, J18, H55.

1. Introduction

Ageing of population in most developed countries has a negative effect on public balances and especially on unfunded or pay-as-you-go (PAYGO) pension systems through additional pension claims that lead to a disequilibrium in dependency rates.

Parametric reforms of pension systems may have negative repercussions on labour markets, while decreasing pension benefits may ultimately lead to rising old-age poverty (Krieger 2014).

It follows that, in the presence of population ageing, it may be necessary to enlarge the contribution base in order to cover existing and new pension claims. This may be expanded through two main channels, either by improving the workers' productivity (and thus their wages) or increasing the contributors' number. In particular, the increase in contributor's number may be obtained recurring to immigration of young foreign labour force.

Razin and Sadka (1999) and Sinn (2000, 2001) propose a simple mechanism showing how immigrants could contribute to pension systems' sustainability independently from their skills. However, these models are based on simplifying assumptions that may lead to some criticism.

In particular, from a welfare-theoretic perspective, unskilled immigration may cause falling wages and productivity or increasing unemployment among native workers (Casarico and Devillanova 2003, Jinno 2011, Kemnitz 2003).

Furthermore, in the longer run, differences in the fertility and skills between immigrants' and natives' descendants may also change the distribution of welfare gains and losses (Krieger 2004). Similarly, repeated unskilled immigration could lower savings, capital per capita and, thus, wages as part of the contribution base (Aslanyan 2014).

There is also a wide welfare literature that criticizes the recourse to low-skilled foreign workers because this solution leads to too large immigrants influx to be accommodated by the host countries (Uebelmesser 2004; Krieger 2005; Serrano, Eguía and Ferreira 2011).

The solution could be a high skill-selective form of immigration that, through higher wages compared to low-skilled immigration, is likely to be more successful due to higher expected contributions (Bonin, Raffelhüschen and Walliser 2000).

Furthermore, immigration has not only the potential to relax demographic strain, but is also likely to induce distributional conflicts among different groups in society.

The public-choice literature usually relies on some version of a ballot (Gaston and Rajaguru 2013) where the median voter (in terms of age or skills) decides on immigrants' amount and skill.

In this models there is a trade-off between costs and benefits, with regard to pension system sustainability, in terms of contributions to the pension system and future pension claims. This trade-off will shape the voting equilibrium resulting from a restriction of immigrants' access or to selective immigration policy (Krieger 2003; Scholten and Thum 1996).

Another implication of the median-voter framework is that today's median voters might allow immigrants into the country, although they might decrease their wages, if this helps to strengthen their political power today and sustain it tomorrow (Haupt and Peters 1998; Sand and Razin 2007; Razin, Sadka and Suwankiri 2010).

The main limit of the public-choice literature cited above is that population ageing is not properly taken into account and is assumed to be static (Haupt and Peters 1998; Sand and Razin 2007; Razin, Sadka and Suwankiri 2010; Lacomba and Lagos 2010). This assumption appears to be a little contradictory, since the problem of the sustainability of pension systems arises as a consequence of changes in the demographic structure of native population either in terms of increased life expectancy or in terms of decreased fertility rates. In fact, the welfare literature usually models the population ageing by means of more articulated hypotheses on the population dynamic (Leers et al 2004, Serrano et al. 2011, Uebelmesser 2004).

The following theoretical model aims to close the gap between the public-choice literature and the welfare-theory. It can be seen as extending Lacomba and Lagos's (2010) framework after changing the assumptions on the population dynamic. Differently from Lacomba and Lagos (2010), here, life expectancy is assumed to be linearly increasing over time and individuals differ in the lifetime

horizon.

The rest of the paper is organized as follows. Section 2 explains the demographic structure of population and the model. Section 3 analyses how the arrival of immigrants affects the welfare of the host population. Section 4 summarizes the main results and provides some policy implications. The appendix includes some computations excluded from the previous sections in order to make the reading more linear.

2. The theoretical framework

The basic theoretical framework is provided by Lacomba and Lagos (2010). One of the core assumptions in their model is that native population is constant over time. We modify this assumption allowing for population ageing due to an increase in life expectancy. Instead, the birth rate is assumed to remain constant. Our assumption better captures the real situation of most industrialized countries, characterized by population ageing and low natality rates. The following subsection describes the demographic dynamic and the theoretical framework of the model.

2.1. Demographic dynamic

We consider an Overlapping Generations Model in continuous time. At each point of time t a new cohort of individuals is born. We assume a constant birth rate, normalized to unity.

In the initial period $t = 0$ there is an uniform and continuous distribution of N individuals on age a , with no uncertainty on the length of their lives, going from zero to T .

Differently from Lacomba and Lagos (2010), we suppose that living agents at period $t = 0$ have different life expectancies. In particular, an individual i of age $a_i = 0$ at time $t = 0$ has a life expectancy E_i equal to

$$E_i = \begin{cases} T + g(a_i) & \text{if } a_i \in [0, T + \Omega], \\ T + \Omega & \text{if } a_i \in (-\infty, 0], \end{cases} \quad (1)$$

where $\Omega \in [0, \frac{T}{4}]$ and $g(a_i) = \Omega - \frac{\Omega}{T}a_i$. For simplicity, we assume that all individuals born after $t = 0$ (individuals with negative age at time $t = 0$) have a life expectancy equal to $T + \Omega$. That is, life expectancy can increase up to a certain limit, the 25% of the current observed life expectancy. This assumption is reasonable in the short run according to data provided by OECD (2009).

As a consequence, there is a progressive increase of population and at period $t = h$, population will also include individuals with age $a \in (T, \mu)$, with $\mu \in [T, \Omega]$.

Furthermore, in the period $t = h$ the amount of old people increases. That is, in the period $t = h$ there is an increase in individuals with age $a \in (T, \mu]$, where $\mu \in (T, T + \Omega]$. In order to quantify the increase in old population in the period $t = h$ we need to distinguish different cases, depending on the future horizon $t = h$ considered.

Case 1: $0 < h < \mu - t$.

Individuals with age a in the period $t = 0$ who are still alive at time $t = h$ are those that will have an age smaller than their life expectancy. By other words it must be satisfied

$$a + h \leq T + g(a). \quad (2)$$

That is,

$$a \leq \lambda(h), \quad (3)$$

where

$$\lambda(h) = T \left(1 - \frac{h}{T + \Omega} \right). \quad (4)$$

We can now compute (4) the increase in old people $\varphi(\mu, h)$ at time $t = h$, that is, the amount of individuals that at time $t = h$ have age $a \in [T, T + \mu]$:

$$\varphi(\mu, h) = \frac{N}{T} \left(h \frac{\Omega}{T + \Omega} \right). \quad (5)$$

Case 2: $\mu - T < h < T$.

The procedure is the same: individuals aged a at time $t = 0$ are still alive at period $t = h$ if equation (2) is verified. Individuals who will have an age $a \in [T, \mu]$ in the period $t = h$ are those who at period $t = 0$ had an age $a \in [T - h, \mu - h]$.

In order to compute the increase in (old) population, we need to distinguish three sub-cases (4).

If $\mu \leq T \left(1 + \frac{\Omega}{T + \Omega} \right)$ and $\mu - T < h < \frac{T + \Omega}{\Omega}(\mu - T)$, then the increase in old people is given by equation (5).

If $\mu \leq T \left(1 + \frac{\Omega}{T + \Omega} \right)$ and $\frac{T + \Omega}{\Omega}(\mu - T) < h < T$, or $\mu > T \left(1 + \frac{\Omega}{T + \Omega} \right)$, then the increase in old people is equal to:

$$\varphi(\mu, h) = \frac{N}{T} (\mu - T). \quad (6)$$

Case 3: $T - h < 0 < \mu - h$.

In this case $T < h < \mu$, so the population at time $t = h$ will include individuals who are not yet born at time $t = 0$. Individuals who will be alive at time $t = h$ are those characterized either by

$$a + h < T + \Omega \quad \text{with} \quad T - h < a < 0; \quad (7)$$

or

$$a < \lambda(h) \quad \text{with} \quad 0 < a < \mu - h. \quad (8)$$

Condition (7) is always satisfied, so the individuals who at time $t = 0$ have an age $a \in [T - h, 0]$ will be all alive at period $t = h$, with $a \in [T, \mu]$. Note that here a is negative so these individuals aren't yet born at period $t = 0$.

The condition (8) leads to distinguish different cases (4). When $T \left(1 + \frac{\Omega}{T + \Omega} \right) < \mu < T + \Omega$, $\lambda(h)$ is smaller than $\mu - h$ and the increase of population is (6);

when $\mu < T \left(1 + \frac{\Omega}{T+\Omega}\right)$, we need to separate two different eventualities: if $T < h < \frac{T+\Omega}{\Omega}(\mu - T)$ we have (5); if $\frac{T+\Omega}{\Omega}(\mu - T) < h < \mu$ we have (6).

Case 4: $T - h < \mu - h < 0$ where $h < T + \Omega$.

In this case, $T < \mu < h < T - \Omega$ and the individuals who will be alive at period $T = h$ (those who have $a + h < T + \Omega$) aren't yet born at time $t = 0$ because they have an age $a \in [T - h, \mu - h]$ in $t = 0$. It follows that in $t = h$ the amount of people aged $a \in [T, \mu]$ will be equal to (6).

Case 5: $T - h < \mu - h < 0$ con $h > T + \Omega$.

In this case, the time horizon is greater than $T + \Omega$, population is now stabilized on $\frac{N}{T}(T + \Omega)$ and the amount of people aged $a \in [T, \mu]$ will be equal to (6), where $\mu = \Omega$.

2.1.1. Demographic structure

The demographic dynamic illustrated above leads to the following demographic structure.

The maximum age a^{max} observed in the population varies with the time horizon $t = h$. In particular with $h \in [0, T + \Omega]$ we have $a^{max} = T + h \frac{\Omega}{T+\Omega}$.

The amount of total population $N(h)$ is function of h too, with $N(h) = \frac{N}{T} \left(T + h \frac{\Omega}{T+\Omega}\right)$. Furthermore the number of individuals with age $a \leq a^*$, where $a^* \leq a^{max}$, is equal to $\frac{N}{T} a^*$.

2.2. Preferences and consumptions

Following Lacomba and Lagos (2010), we normalize the wage rate to unity and the government levies a contribution rate, τ , for a redistributive, pay-as-you-go (PAYGO) pension system in which current workers are net contributors and retired people are net beneficiaries.

Agents have a temporally independent utility function, strictly increasing in instantaneous consumption¹. Let δ be the subjective rate of time preference and c_i^t the individual consumption at time t , the lifetime utility $U_i(c_i^t)$ of an individual i can be written as

$$U_i(a_i, c_i^t) = \int_0^R u(c_i^t) e^{-\delta t} dt + \int_R^{E_i} u(c_i^t) e^{-\delta t} dt, \quad (9)$$

where R indicates the retirement age and $R \in \left[\frac{T}{2}, T\right]^2$. The instantaneous utility function is twice differentiable with $u' > 0$ and $u'' < 0$. Furthermore, the variable c_i^t is the personal consumption at period t and E_i is the life expectancy of the individual i .

The lifetime utility function (9) is subject to the individual budget constraint

$$\int_0^{E_i} c_i^t e^{-rt} dt = \int_0^R (1 - \tau) l_i e^{-rt} dt + \int_R^{E_i} p(t) e^{-rt} dt, \quad (10)$$

¹For simplicity, we exclude leisure from the analysis.

²For the sake of simplicity, following Lacomba and Lagos (2010), we are supposing that initially the number of workers is larger than the number of retirees.

where l_i is the labour productivity of individual i and $p(t)$ the annual pension benefit, function of t . In fact, population ageing increases the number of retired people and influences the per capita amount of pension benefits.

We assume no returns on savings. Furthermore, individuals do not discount the future ($\delta = 0$ and $r = 0$). This assumption implies that each individual will set a constant consumption per period, that is, individuals are fully rational and hence their future income benefits become quantitatively as important as their present income benefits. Moreover, this assumption is adopted in order to avoid myopic individuals (of different ages) ignoring the true impact of current decisions on their pension benefits³.

Furthermore, assuming ($\delta = 0$ and $r = 0$) allows us (Breyer, 1994) to reduce the indirect residual lifetime utility function (9) of an individual i of age a_i to the form

$$U_i(a_i, c_i) \equiv (T + g(a_i)) u(c_i), \quad (11)$$

where c_i is the constant optimal consumption per period (Breyer, 1994) and $T + g(a_i) = E_i$ is the life expectancy, as described in section 2.1.

The constant instantaneous optimal consumption during the individual life is equal to

$$c_i = \frac{1}{T + g(a_i)} (R(1 - \tau)l_i + (T + g(a_i) - R)p(t)). \quad (12)$$

It follows that, during their life, individuals are characterized by different amounts of accumulated wealth, $\pi(a_i, l_i)$, function of their age a_i and l_i , which is given by the total income earned minus total consumption up to the present:

$$\pi(a_i, l_i) = \begin{cases} a_i((1 - \tau)l_i - c_i) & \text{if } a_i \in [0, R], \\ R(1 - \tau)l_i + (a_i - R)p(t) - a_i c_i & \text{if } a_i \in [R, T + g(a_i)]. \end{cases} \quad (13)$$

Note that there exists a threshold labour productivity \bar{l} such that $(1 - \tau)\bar{l} = p(t)$. We assume that $l_i > \bar{l}$. Therefore the accumulated wealth increases linearly with age up to the retirement age, and beyond that age agents start to spend their accumulated savings.

Social security systems may range from a flat-rate pension benefits type (usually referred to as a Beveridgean scheme) to an earnings-related pension benefits type⁴ (usually referred to as a Bismarckian scheme). In this paper we concentrate on a Beveridgean pension system in order to examine the impact of immigration on redistribution. If a Bismarckian pension system were considered, then the effects of immigration would be more beneficial for the native population, since in Bismarckian systems, there is not intra-generational redistribution.

³If δ and r were equal but different from zero, the consumption would not be affected. However, the calculation of pension benefits would be greatly complicated without adding further insights.

⁴See Casamatta et al. (2000) for a more complete classification of pension systems depending on the redistribution nature of the social security system.

Here the pension system is assumed to be unfunded and fully redistributive. It follows that pension benefits depend crucially on amount of retired people. If we consider as time horizon the period $t = h$, on the basis of the demographic structure illustrated above, the amount of retired people at time $t = h$ will be equal to:

$$N_{Ret}(h) = \frac{N}{T} \left(T - R + h \frac{\Omega}{T + \Omega} \right). \quad (14)$$

Consequently, the discounted value of total pension benefits received by an individual i is

$$P = \int_0^{T+g(a_i)-R} \frac{\tau RL}{\left(T - R + \frac{\Omega}{T+\Omega} h \right)} dh, \quad (15)$$

where L is the average productivity of native workers⁵. It is apparent from equation (15) that the time horizon $t = h$ influences the amount of pension benefits and new generations will receive smaller benefits since the retired population increases.

2.3. Immigration's effects

Following Razin and Sadka (1999), we assume that at period $t = 0$ a number $m \in [0, T]$ of immigrants can be allowed in the host country. In the following subsection we will derive the amount of the pension benefits (and the relative effect on consumption) either in the absence or in the presence of immigrants.

Consider the situation of an individual i of age a_i . In the absence of immigration, we can explicit equation (15) as (4):

$$P = \tau RL \frac{T + \Omega}{\Omega} \ln \left(\frac{T + \Omega}{T} - \frac{\Omega^2}{T^2(T + \Omega - R)} a_i \right). \quad (16)$$

We can write the budget constraint in the residual lifetime of an agent aged $a_i \in [0, R]$ as:

$$c_i(T + g(a_i) - a_i) = \begin{cases} \pi(a_i, l_i) + (R - a_i)l_i(1 - \tau) + P, & \text{if } a_i \in [0, R], \\ \pi(a_i, l_i) + P(a_i, 0), & \text{if } a_i \in [R, T + g(a_i)], \end{cases} \quad (17)$$

where c_i is the constant instantaneous consumption (Breyer, 1994) and $P(a_i, 0) = \int_{a_i}^{T+g(a_i)} p(h)dh$.

From equation (17) it follows (4) that we can express the instantaneous consumption of an individual aged $a \in [R, T]$ in the period $t = 0$ as:

$$c_i = \begin{cases} \frac{\pi(a_i, l_i) + (R - a_i)l_i(1 - \tau) + P(a_i)}{T + g(a_i) - a_i} & \text{if } a_i \in [0, R], \\ \frac{\pi(a_i, l_i) + \psi(a_i)}{T + g(a_i) - a_i} & \text{if } a_i \in [R, T + g(a_i)], \end{cases} \quad (18)$$

⁵Equation (15) comes from $P = \int_R^{T+g(a_i)} p(h)dh = \int_R^{T+g(a_i)} \frac{\frac{N}{T} \tau RL}{\left(T - R + \frac{\Omega}{T+\Omega} h \right)} dh$.

where $\psi(a_i) = \tau RL \frac{T+\Omega}{\Omega} \ln \left(\frac{T+(T+\Omega)^2 - RT(T+\Omega) - \Omega^2 a_i}{((T+\Omega)(T-R) + \Omega a_i)T} \right)$.

Now we consider the case that, at period $t = 0$, a number $m \in [0, T]$ of immigrants is allowed in the host country. Following Razin and Sadka (1999), we assume that immigrants are all young with $a_j = 0 \forall j \in [0, m]$, low-skilled and with same preferences and fertility rate of natives.

It follows that, starting from period $t = 0$, a new cohort of $\frac{N}{T}(1+m)$ individuals born in each instant. It follows that the amount of immigrants of age $a \leq h$ at time $t = h$ is $\frac{N}{T}mh$ if $h < R$ or $\frac{N}{T}mR$ if $h > R$.

In the presence of immigration, the total amount of contribution $\bar{Z}(m, h)$ raised from workers is

$$\bar{Z}(m, h) = \begin{cases} \frac{N}{T}\tau RL + \frac{N}{T}\tau mhI & \text{if } h \in [0, R), \\ \frac{N}{T}\tau RL + \frac{N}{T}\tau mRI & \text{if } h \in [R, \infty), \end{cases} \quad (19)$$

where I is the average productivity of immigrants.

The total amount of retired people in the presence of immigrants at time $t = h$ is equal to the sum of native retirees and immigrant retirees.

The native retired people, denoted by N_{Ret}^N , are

$$N_{Ret}^N = \begin{cases} \frac{N}{T} \left(T - R + \frac{\Omega}{T + \Omega} h \right) & \text{if } h \in [0, T + \Omega), \\ \frac{N}{T} (T + \Omega - R) & \text{if } h \in [T + \Omega, \infty). \end{cases} \quad (20)$$

The amount of immigrant retired people, N_{Ret}^I , is instead

$$N_{Ret}^I = \begin{cases} 0 & \text{if } h \in [0, R), \\ \frac{N}{T} (h - R) m & \text{if } h \in [R, T + \Omega), \\ \frac{N}{T} (T - R + \Omega) m & \text{if } h \in [T + \Omega, \infty). \end{cases} \quad (21)$$

3. Welfare analysis through pension benefits

For the sake of simplicity we consider a temporal horizon smaller than the maximum life expectancy ($h \leq T + \Omega$) at time $t = 0$. It follows that, in the presence of immigrants, the amount of pension benefit at time h will be equal

to⁶

$$p(h, m) = \begin{cases} \frac{\tau(RL + mhI)}{\left(T - R + \frac{\Omega}{T + \Omega}h\right)} & \text{if } 0 \leq h \leq R, \\ \frac{\tau(RL + mRI)}{\left(T - R + \frac{\Omega}{T + \Omega}h\right) + m(h - R)} & \text{if } R \leq h \leq T + \Omega. \end{cases} \quad (22)$$

In the absence of immigration, the amount of a pension benefit perceived by each pensioner at time $t = h$ is equal to

$$p(h) = \frac{\tau RL}{T - R + \frac{\Omega}{T + \Omega}h}. \quad (23)$$

Now we can study the effect of immigration on the welfare of native individuals through the effect of immigration on their pension benefits.

Let $P(a_i, m)$ the total pension benefits that an individual of age a_i at time $t = 0$ in the presence of immigrants,

$$P(a_i, m) = \begin{cases} \int_{R-a_i}^{T+g(a_i)-a_i} p(h, m)dh, & \text{if } 0 \leq a_i \leq R, \\ \int_0^{T+g(a_i)-a_i} p(h, m)dh, & \text{if } R \leq a_i \leq T, \end{cases} \quad (24)$$

immigration will increase the individual welfare if

$$\Delta = \int_{\max(0, R-a_i)}^{T+g(a_i)-a_i} (p(h, m) - p(h))dh \quad (25)$$

is greater than zero.

We need to distinguish different cases.

Case 1 - retirees: $a_i \in [R, T]$.

In this case we consider the welfare of individuals who are already retired⁷ at time $t = 0$. It follows that equation (25) takes the form:

$$\Delta = \int_0^{T+g(a_i)-a_i} (p(h, m) - p(h))dh. \quad (26)$$

⁶If $h > T + \Omega$, then $p(h, m) = \frac{\frac{N}{T}\tau(RL+mIR)}{\frac{N}{T}(T-R+\Omega) + \frac{N}{T}m(T-R+\Omega)}$.

⁷Note that, since we assume $\Omega < \frac{T}{4}$, there are no retirees who share pensions with immigrants.

In this case $T + g(a_i) - a_i < R$, i.e. $a_i > \frac{T(T+\Omega-R)}{T+R}$, so all retired individuals will not share the retirement period with immigrants. It follows that equation (26) takes the form:

$$\Delta = \Delta_R = \int_0^{T+g(a_i)-a_i} \frac{\tau m h I}{\left(T - R + \frac{\Omega}{T+\Omega} h\right)} dh. \quad (27)$$

From equation (27), it is apparent that $\Delta_R > 0$; $\frac{\partial \Delta_R}{\partial m} > 0$ and $\frac{\partial \Delta_R}{\partial I} > 0$. The following proposition can therefore be stated.

Proposition 1. *In the presence of a PAYGO pension system and linear increase in life expectancy, for any native retired individual aged $a_i \in [R, T]$, any immigrant quota $m > 0$ increases the pension benefits, regardless of immigrants' labour productivity.*

Proof. This follows straightforwardly from (27). □

Case 2 - the "non-sharing pension" workers: $a_i \in \left[\frac{T(T+\Omega-R)}{T+R}, R\right)$.

In this case individuals are still workers when immigrants enter in the system at period $t = 0$. It follows that the expression (25) takes the form:

$$\Delta = \int_{R-a_i}^{T+g(a_i)-a_i} (p(h, m) - p(h)) dh. \quad (28)$$

Furthermore, since $a_i > \frac{T(T+\Omega-R)}{T+R}$, than these workers will not share the retirement period with immigrants and equation (28) takes the form:

$$\Delta = \Delta_{NSW} = \int_{R-a_i}^{T+g(a_i)-a_i} \frac{\tau m h I}{\left(T - R + \frac{\Omega}{T+\Omega} h\right)} dh. \quad (29)$$

From equation (29), it is apparent that $\Delta_{NSW} > 0$; $\frac{\partial \Delta_{NSW}}{\partial m} > 0$ and $\frac{\partial \Delta_{NSW}}{\partial I} > 0$.

The following proposition can therefore be stated.

Proposition 2. *In the presence of a PAYGO pension system and linear increase in life expectancy, for any native "old" worker aged $a_i \in \left[\frac{T(T+\Omega-R)}{T+R}, R\right)$, any immigrant quota $m > 0$ increases the pension benefits, regardless of immigrants' labour productivity.*

Proof. This follows straightforwardly from (29). □

Case 3: the "sharing pension" workers $a_i \in \left[0, \frac{T(T+\Omega-R)}{T+R}\right)$.

In this case workers will share the retirement period with immigrant retirees and equation (28) takes the form:

$$\begin{aligned} \Delta = \Delta_{SW} = & \int_{R-a_i}^R \frac{\tau m h I}{\left(T - R + \frac{\Omega}{T+\Omega} h\right)} dh \\ & + \int_R^{T+g(a_i)-a_i} \left(\frac{\tau(RL + mRI)}{\left(T - R + \frac{\Omega}{T+\Omega} h\right) + m(h - R)} - \frac{\tau RL}{\left(T - R + \frac{\Omega}{T+\Omega} h\right)} \right) dh. \end{aligned} \quad (30)$$

The first component of the RHS in (30) represents the temporary increase in pension benefits received before the first generation of immigrants reaches the retirement period. As in the previous cases, native individuals will benefit during these years from the arrival of immigrants. The second component includes pension benefits received once immigrants start to enter the retirement period. In this second component, besides the usual positive effect on the native population's pension benefits derived from immigrants' additional contributions, we also observe a negative effect, due to the larger number of retirees. Therefore the final effect on pension benefits is ambiguous and will depend on the other parameters.

In particular, defining \bar{I} as the threshold value of the immigrants' average productivity, we can summarize the possible results in the following proposition.

Proposition 3. *In the presence of a PAYGO pension system and linear increase in life expectancy, for any native "young" worker aged $a_i \in \left[0, \frac{T(T+\Omega-R)}{T+R}\right]$, and for any immigrant quota $m > 0$, there exists a threshold average labour productivity of immigrants, such that $\Delta_{NSW} > 0$*

Proof. See 4. □

Note that, since we assume $\Omega < \frac{T}{4}$, the "sharing pension" workers represents less than the 50% of native population and consequently a majority of the population, formed by older workers and retirees, would benefit from an open borders policy.

4. Final remarks

In most developed countries public pension systems are suffering from the ageing of population. Immigration could mitigate the financial problems of the public pension systems.

With an Overlapping Generations Model in continuous time we have investigated the impact that low-skill immigration has on the domestic population's welfare in the presence of increasing life expectancy.

Assuming that life expectancy increases less than 25% during one generation, most of the current native cohorts (retirees and older workers) gain from immigration. On the other hand, younger workers, who will coincide with immigrants in their retirement periods, will either gain or lose from immigration depending on the average labour productivity of the immigrants. If immigrant's

labour productivity is higher than a threshold value, immigration might entail a Pareto improvement for the whole native population.

Consequently, as in Leers et al. (2004), while some young native generations may not welcome immigration and prefer a policy of closed borders, others may get benefits from immigrants and prefer the opposite policy.

Assuming that life expectancy increases less than 25%, seems to be reasonable for developed countries, according to OECD (2009).

However, demographic statistics on long time periods, show a dramatic increase in average life expectancy during the 20th century. Life expectancy at birth now exceeds 83 years in Japan and is at least 81 years in several other countries. Less developed regions of the world have also experienced a steady increase in life expectancy since World War II. A notable exception is the fall in life expectancy in many parts of Africa because of HIV outbreak.

The most dramatic and rapid gains have occurred in East Asia, where life expectancy at birth increased from less than 45 years in 1950 to more than 74 years today, that is, more than 60% (WHO 2011).

It follows that many countries are experiencing or will experience in the future (if public health conditions will improve) increases in life expectancy higher than 25%. Our results show that in the presence of large increase in life expectancy and low-skilled migration the welfare of the majority of native population (with regard to pension benefits) decreases. In fact, in this case the “sharing pension” workers represents more than the 50% of native population and consequently a majority. It follows that immigration can not be politically sustainable without further parametric reforms. In particular, after increasing the retirement age R , the quota of the “sharing pension” workers decreases and they will no longer represent the majority of population. As a consequence, an open borders policy would improve pension benefits of most of native population, leading them to higher welfare levels.

To conclude, immigration is likely to induce distributional conflict between different groups in society. The increasing number of contributors due to immigration will result in higher pension benefits for both retirees and older workers. Future immigrants’ pension claims imply that younger workers will either gain or lose from immigration depending on the immigrants’ labour productivity.

In the case of small expected increase in life expectancy, immigration increases the welfare of the majority of native population, given by retirees and older workers. This is likely to be the case of developed countries, which have already experienced a great “jump” in life expectancy in the last century.

Furthermore, in the presence of expected high increases in life expectancy, immigration may affect negatively the welfare of the majority of native population in the absence of further parametric reforms of the pension system. This is the case of developing countries that in some cases are experiencing increases in life expectancy higher than the 50% of population, according to WHO (2011).

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Appendix A

We need to compute the increase in old people, given by the amount of individuals that at time $t = h$ have age $a \in [T, T + \mu]$, where $\mathbf{0} < \mathbf{h} < \mu - \mathbf{t}$.

By other words, we need to find the amount of people who at time $t = 0$ had an age $a \in [T - h, T]$ and who are still alive after a period h . It is useful to distinguish between two sub-cases. In the first sub-case

$$\lambda(h) < T - h. \quad (.1)$$

In the second sub-case

$$\lambda(h) \geq T - h. \quad (.2)$$

Substituting equation (4) in (.1) we obtain

$$h \frac{\Omega}{T + \Omega} < 0, \quad (.3)$$

that is impossible because $h > 0$.

It follows that, when $0 < h < \mu - t$ the unique sub-case admissible is (.2). As a consequence, the increase $\varphi(\mu, h)$ in the old population at time $t = h$ is equal to:

$$\varphi(\mu, h) = \frac{N}{T} [\lambda(h) - (T - h)]. \quad (.4)$$

Substituting equation (4) into (.4) we obtain:

$$\varphi(\mu, h) = \frac{N}{T} \left(h \frac{\Omega}{T + \Omega} \right). \quad (.5)$$

Appendix B

In this case $\mu - \mathbf{T} < \mathbf{h} < \mathbf{T}$. We proceed in the same way as in 4: the condition for which an individual aged a at time $t = 0$ is still alive at period $t = h$ remains the (2). Individuals who will have an age $a \in [T, \mu]$ in the period $t = h$ are those who at period $t = 0$ have an age $a \in [T - h, \mu - h]$.

From 4 we know that equation (.2) is always valid. However, in this case we also need to control if $\lambda(h)$ is greater or smaller than $\mu - h$. In particular

$$\lambda(h) < \mu - h \quad (.6)$$

when

$$h < \frac{T + \Omega}{\Omega} (\mu - T). \quad (.7)$$

Furthermore, in this case we have that $h < T$, so we must have that $\frac{T + \Omega}{\Omega} (\mu - T) \leq T$. That is,

$$\mu \leq T \left(1 + \frac{\Omega}{T + \Omega} \right). \quad (.8)$$

If (.8) is satisfied there are two eventualities. The first is

$$\mu - T < h < \frac{T + \Omega}{\Omega} (\mu - T), \quad (.9)$$

that leads to $\lambda(h) < T - h$. In this case $\varphi(\mu, h) = \frac{N}{T} (\lambda(h) - (T - h))$ and (5) is verified.

The second is

$$\frac{T + \Omega}{\Omega}(\mu - T) < h < T, \quad (.10)$$

that leads to $\lambda(h) > \mu - h$. In this case the amount of individuals at period $t = h$ with an age $a \in [T, T + \mu]$ will be equal to

$$\varphi(\mu, h) = \frac{N}{T} (\mu - T). \quad (.11)$$

We consider now the situation:

$$\mu > T \left(1 + \frac{\Omega}{T + \Omega} \right). \quad (.12)$$

In this case $\frac{T + \Omega}{\Omega}(\mu - T) > T$ and we must consider the individuals who at time $t = 0$ were aged $a \in [\mu - T, T]$. Also in this case, the amount of individuals who at period $t = h$ have an age $a \in [\mu - T, T]$ is still equal to (.11).

Appendix C

The condition (8) leads to distinguish the case where $\lambda(h)$ is smaller or greater than $\mu - h$:

$$\lambda(h) < \mu - h \Leftrightarrow h < \frac{T + \Omega}{\Omega}(\mu - T). \quad (.13)$$

Since, for assumption, it is $T < h < \mu$, so also need to check where is the 'position' of expression $\frac{T + \Omega}{\Omega}(\mu - T)$ in the range $[T, \mu]$. We know that:

$$\frac{T + \Omega}{\Omega}(\mu - T) < T \Leftrightarrow \mu < T \left(1 + \frac{\Omega}{T + \Omega} \right); \quad (.14)$$

$$\frac{T + \Omega}{\Omega}(\mu - T) < \mu \Leftrightarrow \mu < T + \Omega. \quad (.15)$$

So two subcases are possible. In the first case, when $T \left(1 + \frac{\Omega}{T + \Omega} \right) < \mu < T + \Omega$, $\lambda(h)$ is smaller than $\mu - h$, so the (6) is verified. In the second one, when $\mu < T \left(1 + \frac{\Omega}{T + \Omega} \right)$, we need to separate two different eventualities: if $T < h < \frac{T + \Omega}{\Omega}(\mu - T)$ then $\lambda(h) < \mu - h$, so the result is equation (5); if $\frac{T + \Omega}{\Omega}(\mu - T) < h < \mu$ the result is equation (6).

Appendix D

From equation (15) we have:

$$P(a_i) = \int_R^{T+g(a_i)} \frac{\frac{N}{T} \tau RL}{\frac{N}{T} \left(T - R + \frac{\Omega}{T + \Omega} h \right)} dh. \quad (.16)$$

Equation (.16) can be written as:

$$P(a_i) = \tau RL \int_R^{T+g(a_i)} \frac{1}{\left(T - R + \frac{\Omega}{T+\Omega}h\right)} dh. \quad (.17)$$

Furthermore, we have that:

$$\int \frac{1}{\left(T - R + \frac{\Omega}{T+\Omega}h\right)} dh = \frac{T + \Omega}{\Omega} \ln \left(T - R + \frac{\Omega}{T - \Omega}h \right). \quad (.18)$$

Expressing

$$F(h) = \frac{T + \Omega}{\Omega} \ln \left(T - R + \frac{\Omega}{T - \Omega}h \right), \quad (.19)$$

and remembering that $g(a_i) = \Omega - \frac{\Omega}{T} a_i$, the final expression will be:

$$P(a_i) = \int_R^{T+g(a_i)} p(h)dh = \tau RL (F(T + g(a_i)) - F(R)), \quad (.20)$$

that is,

$$P(a_i) = \tau RL \frac{T + \Omega}{\Omega} \left(\ln \left(T - R + \frac{\Omega}{T + \Omega} \left(T + \Omega - \frac{\Omega}{T} a_i \right) \right) - \ln \left(T - R + \frac{\Omega}{T + \Omega} R \right) \right). \quad (.21)$$

Simplifying the (.21) we obtain:

$$P(a_i) = \tau RL \frac{T + \Omega}{\Omega} \ln \left(\frac{T + \Omega}{T} - \frac{\Omega^2}{T^2(T + \Omega - R)} a_i \right). \quad (.22)$$

Appendix E

Substituting equation (16) in (17) we can express the instantaneous consumption of an individual of age $a \in [R, T]$ in the period $t = 0$ as:

$$c_i = \frac{1}{T + g(a_i) - a_i} (\pi(a_i, l_i) + (R - a_i)l_i(1 - \tau) + P(a_i)). \quad (.23)$$

Furthermore, from equation (17) we have that consumption in residual lifetime of an individual aged $a \in [R, T]$ at period $t = 0$ must satisfy the constraint:

$$c_i(T + g(a_i) - a_i) = \pi(a_i, l_i) + \int_{a_i}^{T+g(a_i)} p(h)dh. \quad (.24)$$

It's easy to prove that

$$\int_{a_i}^{T+g(a_i)} p(h)dh = \tau RL (F(T + g(a_i)) - F(a_i)), \quad (.25)$$

where $F()$ corresponds to equation .19. We can rearrange equation (.25) as:

$$\int_{a_i}^{T+g(a_i)} p(h)dh = \tau RL \frac{T-\Omega}{2} \left(\ln \left(T-R + \frac{\Omega}{T+\Omega} \left(T+\Omega - \frac{\Omega}{T} a_i \right) \right) - \ln \left(T-R + \frac{\Omega}{T+\Omega} a_i \right) \right). \quad (.26)$$

Simplifying:

$$\int_{a_i}^{T+g(a_i)} p(h)dh = \tau RL \frac{T-\Omega}{2} \ln \left(\frac{T(T+\Omega)^2 - RT(T+\Omega) - \Omega^2 a_i}{T((T+\Omega)(T-R) + \Omega a_i)} \right). \quad (.27)$$

The total lifetime consumption of an individual aged $a_i \in [R, T]$ at period $t = 0$ will be equal to:

$$c_i(T + g(a_i) - a_i) = \pi(a_i, l_i) + \tau RL \frac{T-\Omega}{2} \ln \left(\frac{T(T+\Omega)^2 - RT(T+\Omega) - \Omega^2 a_i}{T((T+\Omega)(T-R) + \Omega a_i)} \right). \quad (.28)$$

Let $\psi(a_i) = \tau RL \frac{T+\Omega}{\Omega} \ln \left(\frac{T+(T+\Omega)^2 - RT(T+\Omega) - \Omega^2 a_i}{((T+\Omega)(T-R) + \Omega a_i)T} \right)$, the (.28) can be written as:

$$c_i = \frac{1}{T + g(a_i) - a_i} (\pi(a_i, l_i) + \psi(a_i)). \quad (.29)$$

Appendix F

For simplicity, we provide a proof of proposition 3 in the less favourable case for native workers, that is, when $a_i = 0$ and native workers will share the full retirement periods with immigrants. The proof for other cases (less stringent) where $a_i \in \left(0, \frac{T(T+\Omega-R)}{T+R}\right)$ can be straightforwardly derived following the same procedure.

Considering $a_i = 0$, we can write equation (30) as:

In this case workers will share the retirement period with immigrant retired people and equation (28) takes the form:

$$\begin{aligned} \Delta = \Delta_{SW} &= \int_R^{T+\Omega} \frac{\tau(RL + mRI)}{\left(T-R + \frac{\Omega}{T+\Omega}h\right) + m(h-R)} dh \\ &\quad - \int_R^{T+\Omega} \frac{\tau RL}{\left(T-R + \frac{\Omega}{T+\Omega}h\right)} dh. \end{aligned} \quad (.30)$$

From 4, it follows that the second component of the RHS of equation (.30) is:

$$\int_R^{T+\Omega} \frac{\tau RL}{\left(T-R + \frac{\Omega}{T+\Omega}h\right)} dh = \tau RL \frac{T+\Omega}{\Omega} \ln \left(\frac{T+\Omega}{T} \right). \quad (.31)$$

The first component of the RHS of equation (.30) is equal to:

$$\tau(RL + mRI) \int_R^{T+\Omega} \frac{1}{T - R + \frac{\Omega}{T+\Omega}h + m(h - R)} dh. \quad (.32)$$

Furthermore, we have that:

$$\int \frac{1}{T - R + \frac{\Omega}{T+\Omega}h + m(h - R)} dh = Z(h), \quad (.33)$$

where $Z(h) = \frac{T + \Omega}{(1 + m)\Omega + mT} \ln \left(T - (1 + m)R + \frac{(1+m)\Omega + mT}{T+\Omega}h \right)$

It follows that

$$\int_R^{T+\Omega} \frac{\tau(RL + mRI)}{\left(T - R + \frac{\Omega}{T+\Omega}h \right) + m(h - R)} dh = \tau(RL + mRI)(Z(T + \Omega) - Z(R)). \quad (.34)$$

That is,

$$\int_R^{T+\Omega} \frac{\tau(RL + mRI)}{\left(T - R + \frac{\Omega}{T+\Omega}h \right) + m(h - R)} dh = \quad (.35)$$

$$\tau(RL + mRI) \frac{T + \Omega}{(1 + m)\Omega + mT} \ln \left(\frac{(1 + m)(T + \Omega)}{T} \right).$$

Substituting equations (.31) and (.35) in equation (.30) and solving $\Delta_{SW} > 0$ we obtain that $I > \bar{I}$, where:

$$\bar{I} = \frac{L(m(T + \Omega) + \Omega) \ln \left(\frac{T+\Omega}{\Omega} \right)}{m\Omega \ln \left(\frac{(1+m)(T+\Omega)}{T} \right)} - \frac{L}{m} \quad (.36)$$